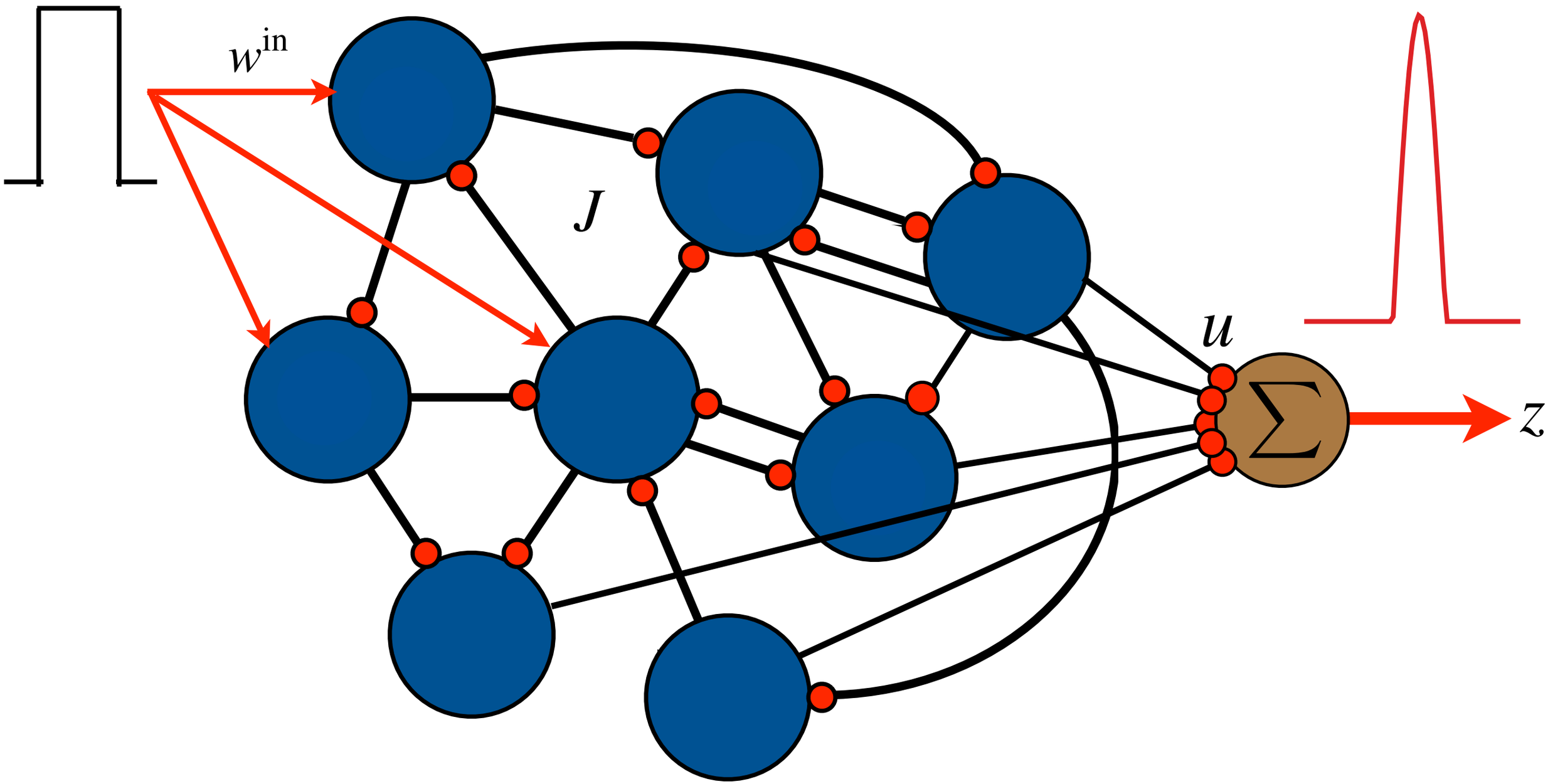
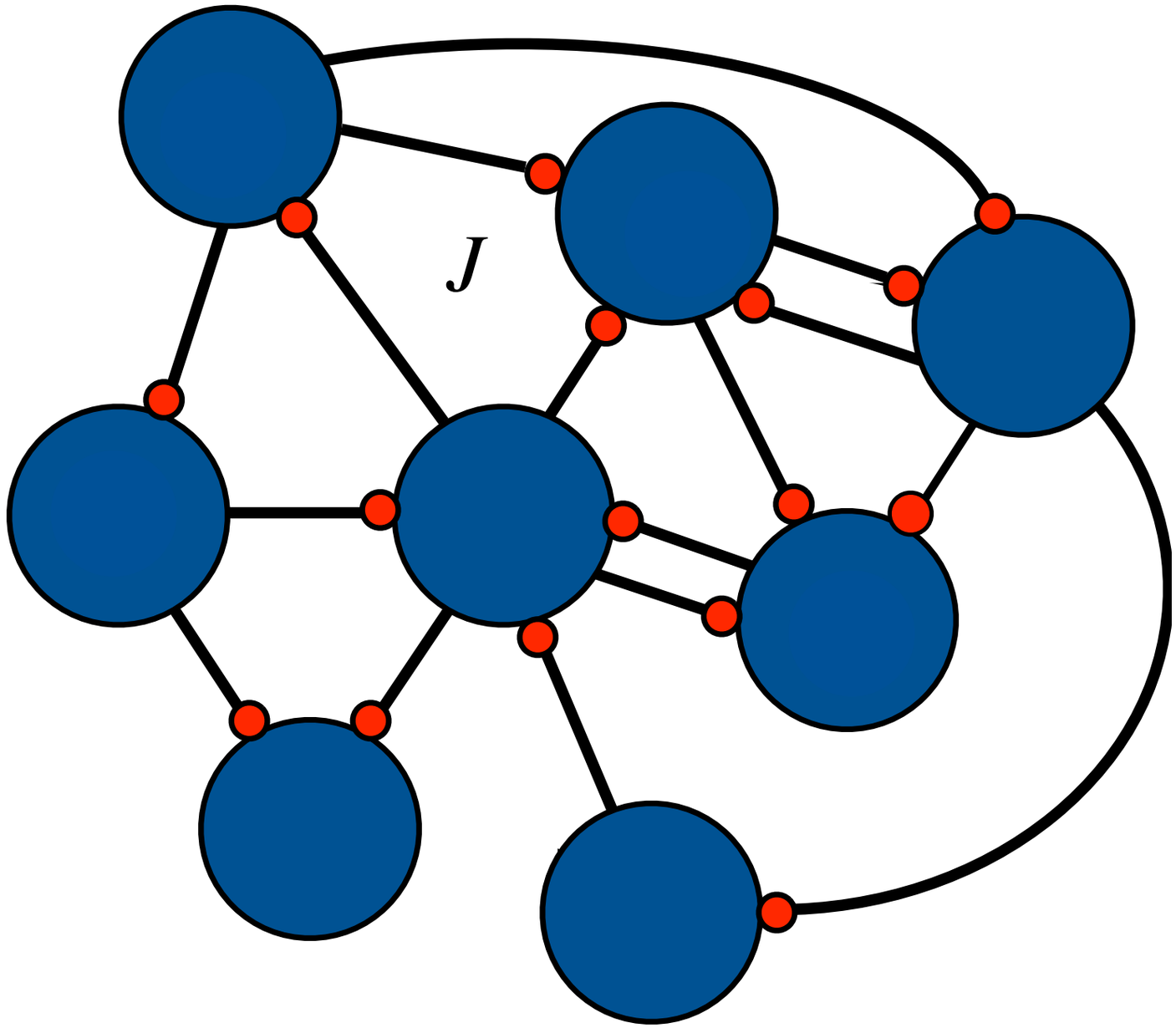


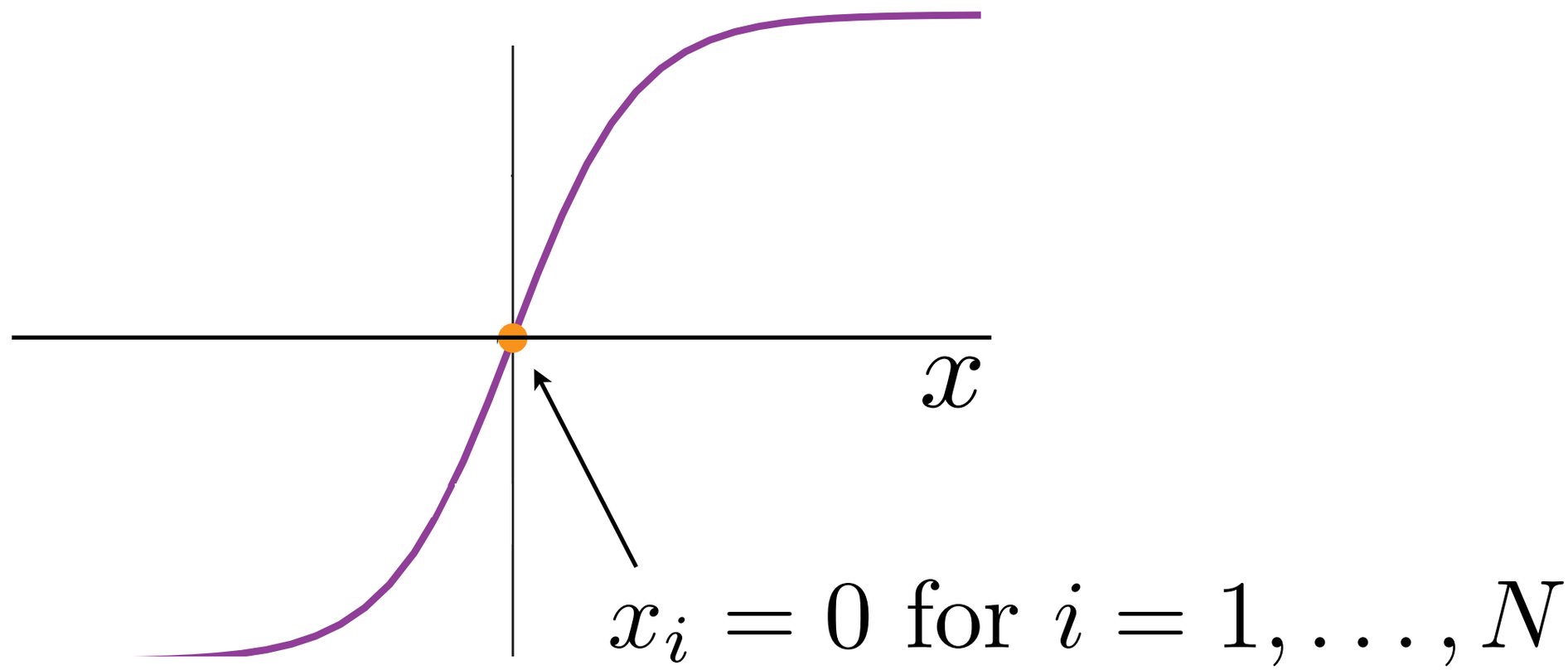
$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} G(x_j) + w_i^{\text{in}} I(t)$$

$$z = \sum_{i=1}^{N} u_i G(x_i) \qquad G(x) = \tanh(x)$$



$$\tau \frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} \tanh(x_j)$$

$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} \tanh(x_j)$$



$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} \tanh(x_j)$$

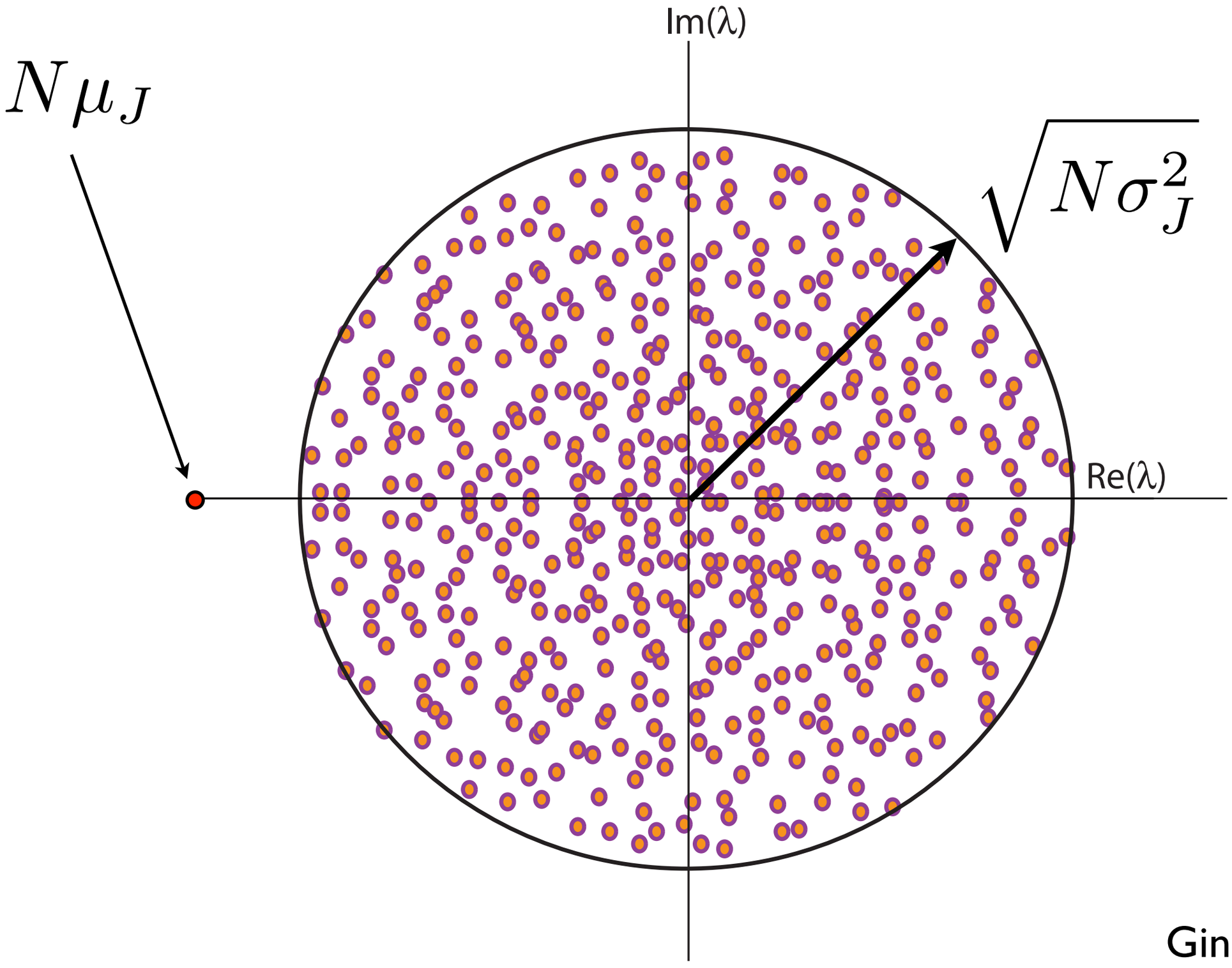
$$x_i = 0 \text{ for } i = 1, \ldots, N$$

stability

$$M_{ij} = -\delta_{ij} + J_{ij}$$

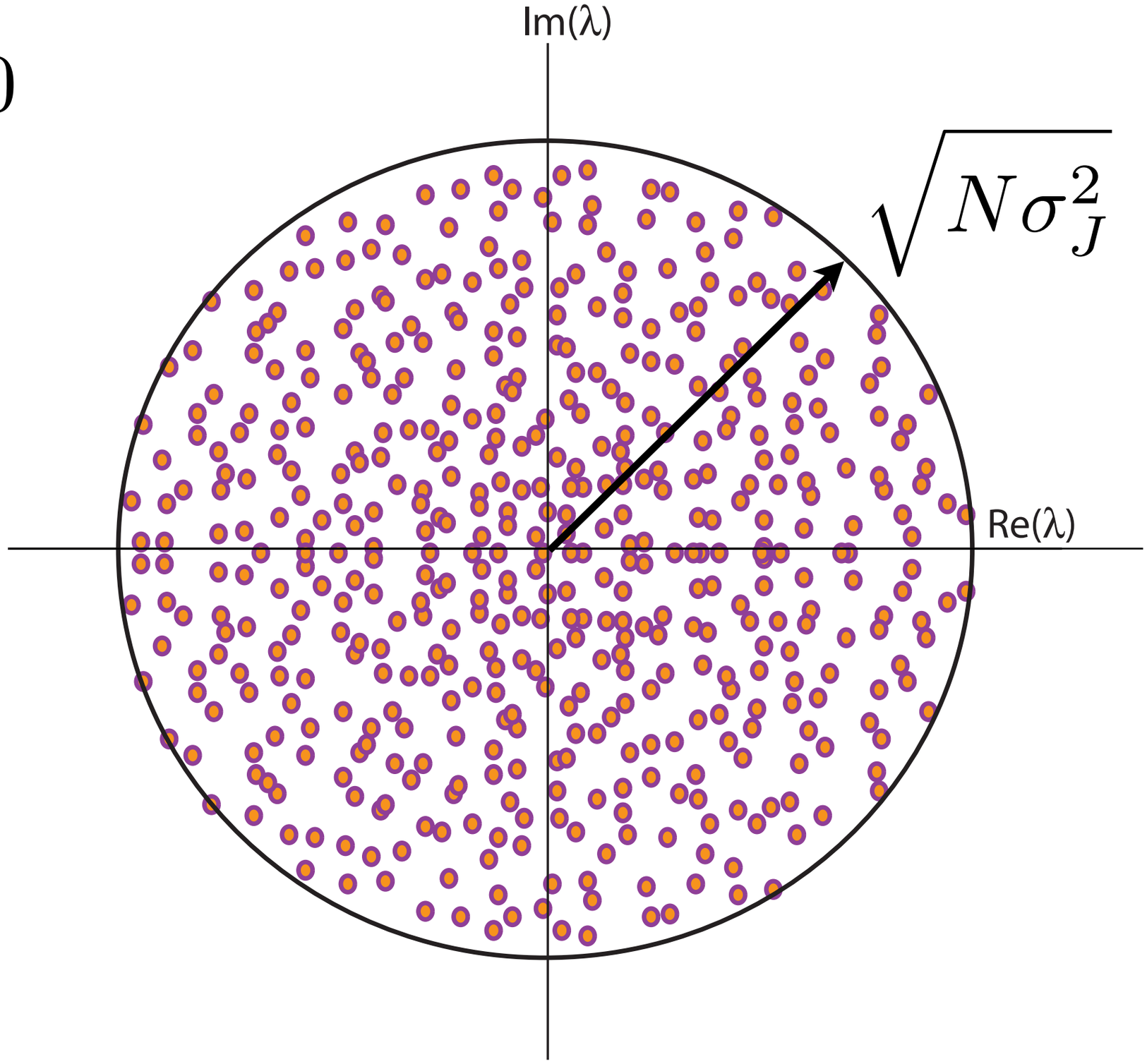
compute eigenvalues

$$\langle J_{ij} \rangle = \mu_J \quad \text{and} \quad \langle J_{ij}^2 \rangle = \sigma_J^2$$



Ginibre, 1965
Girko, 1985
Tao et al., 2010

$$\mu_J = 0$$

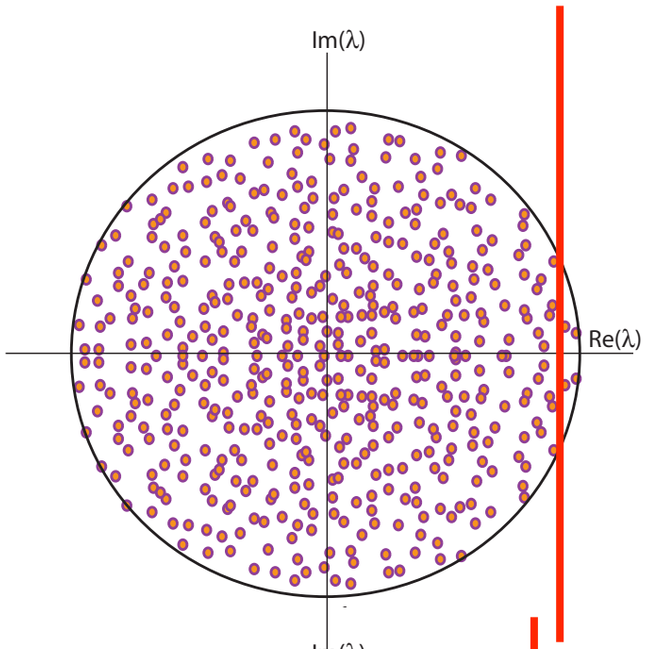


$$\sqrt{N\sigma_J^2} = g$$

fixed point

x

time

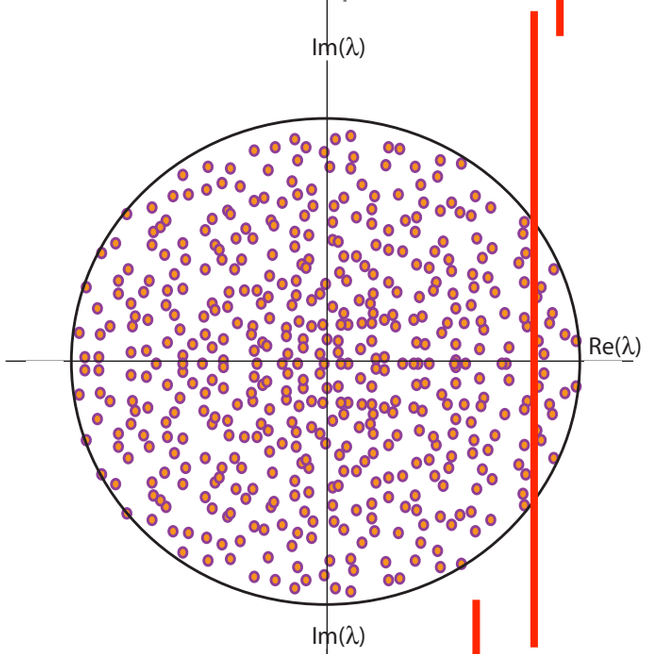


oscillation

x

5
10
15
20
25

time

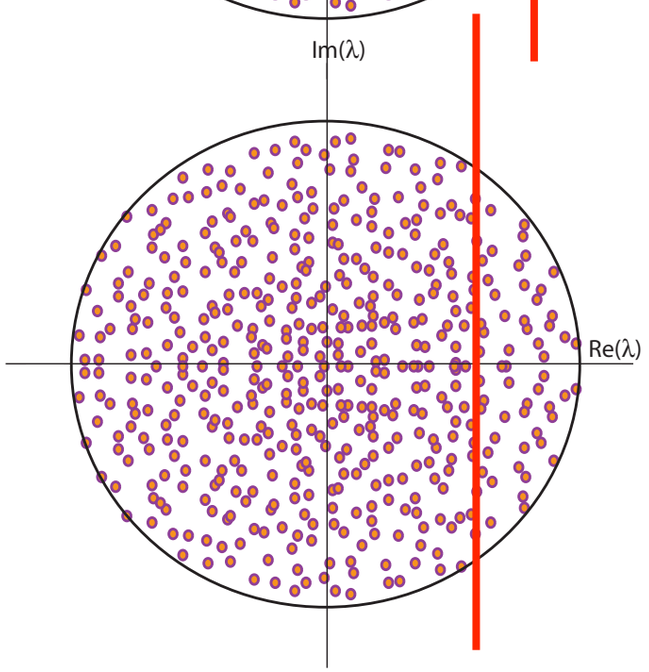


chaos

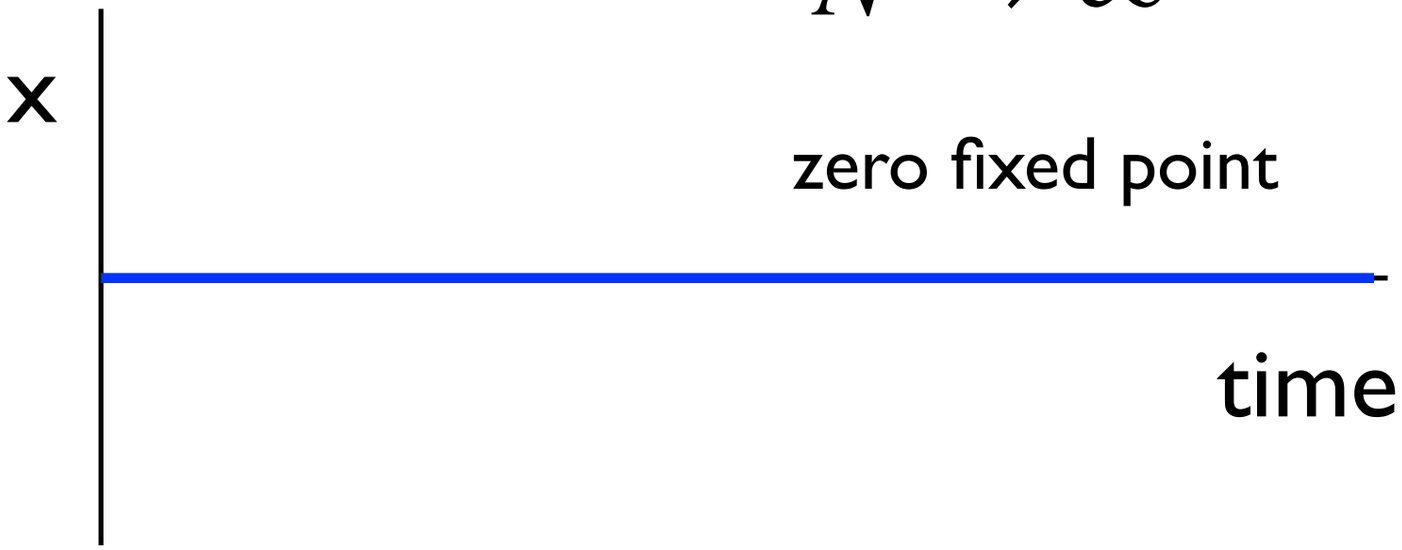
x

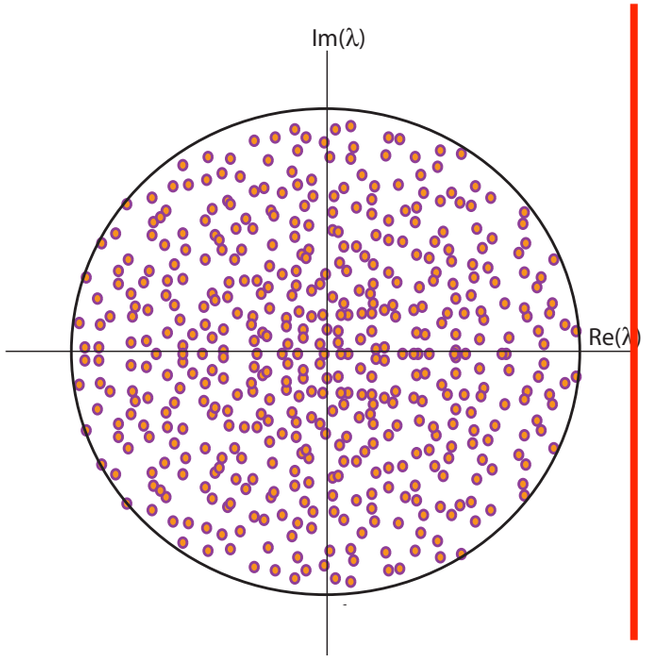
4
6
8
0

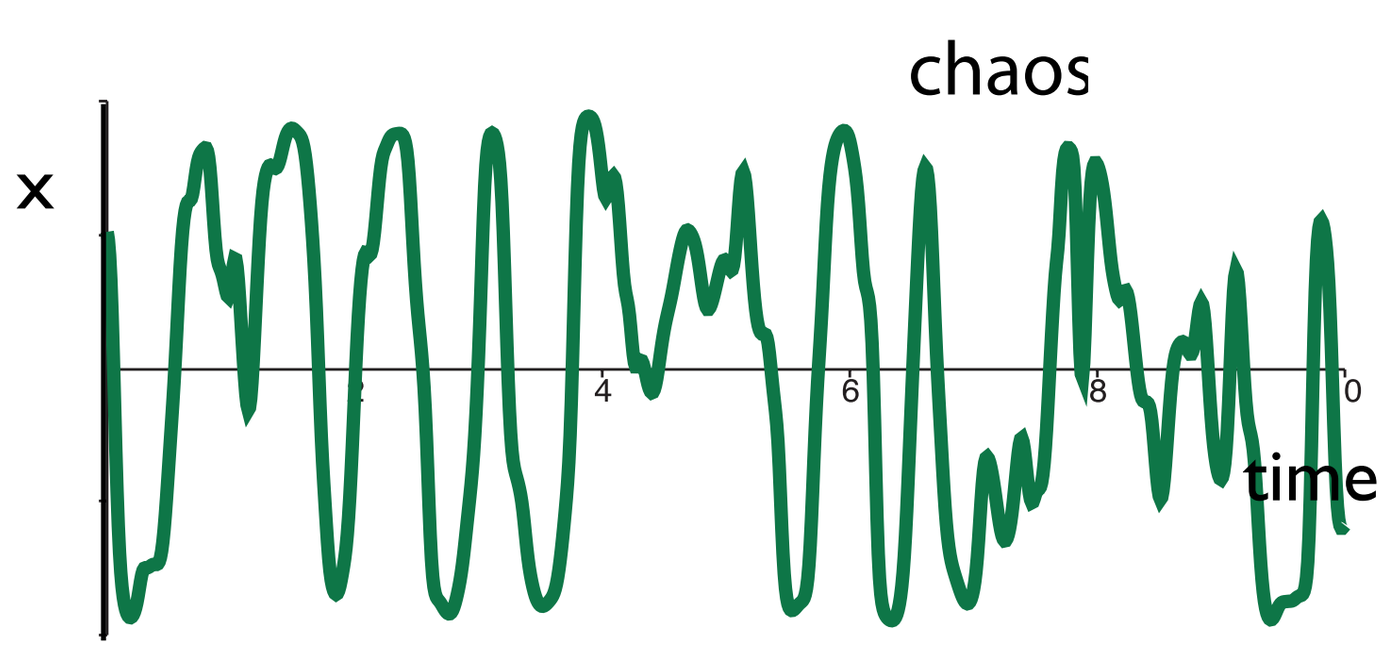
time

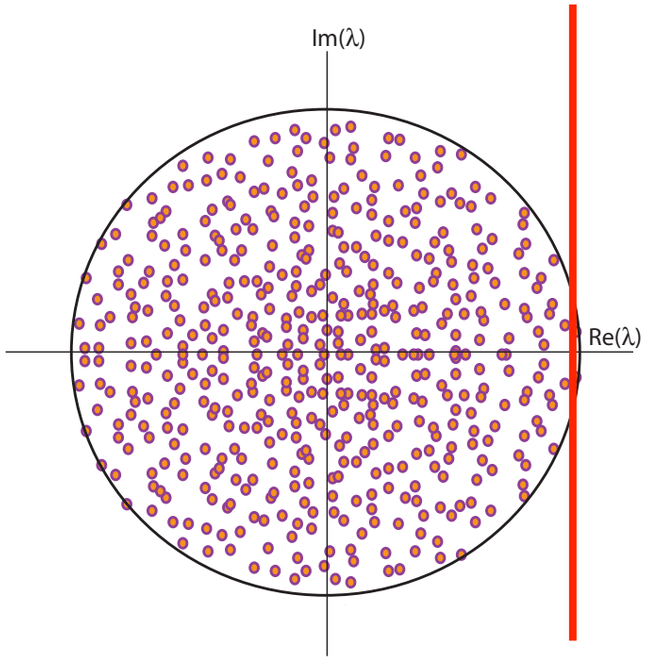


$$N \longrightarrow \infty$$

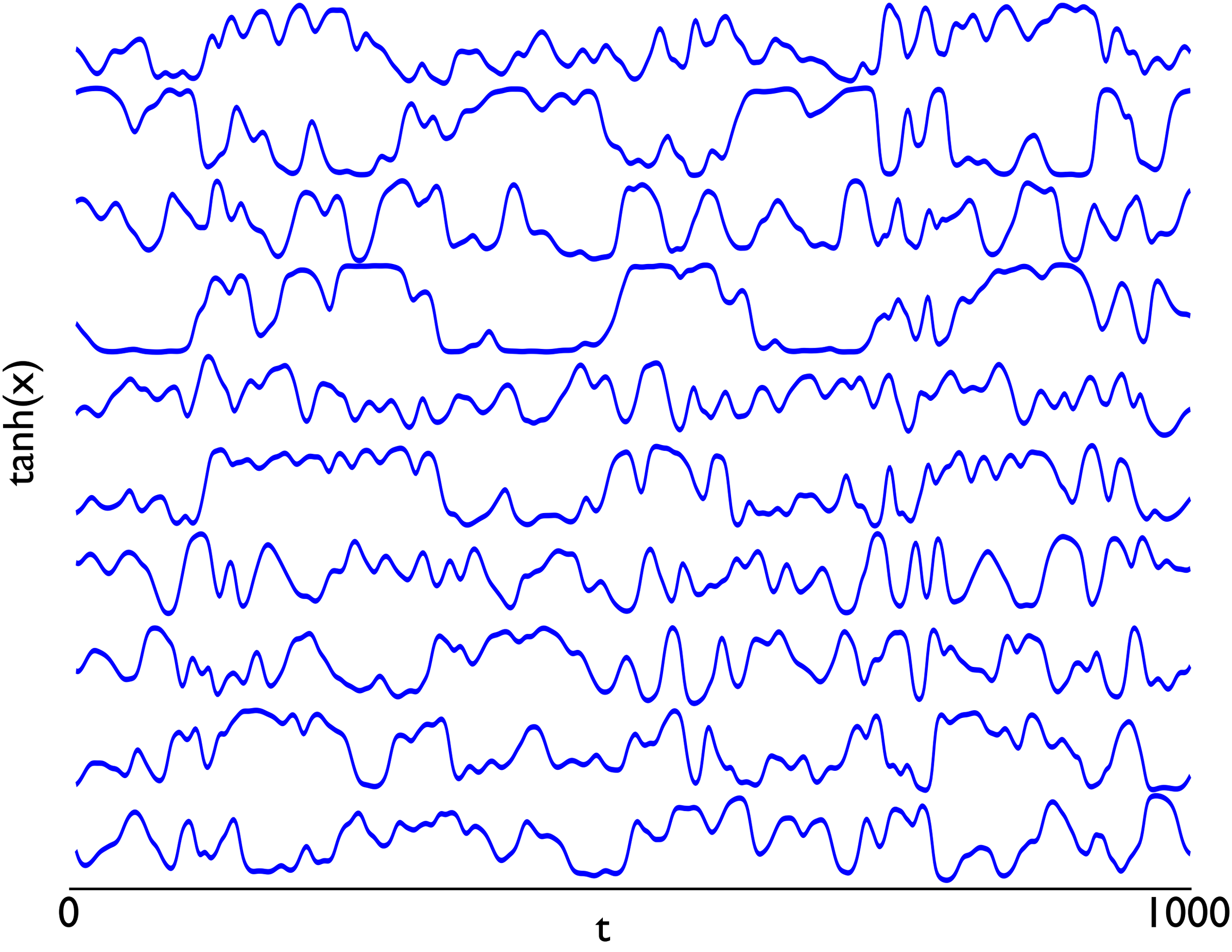




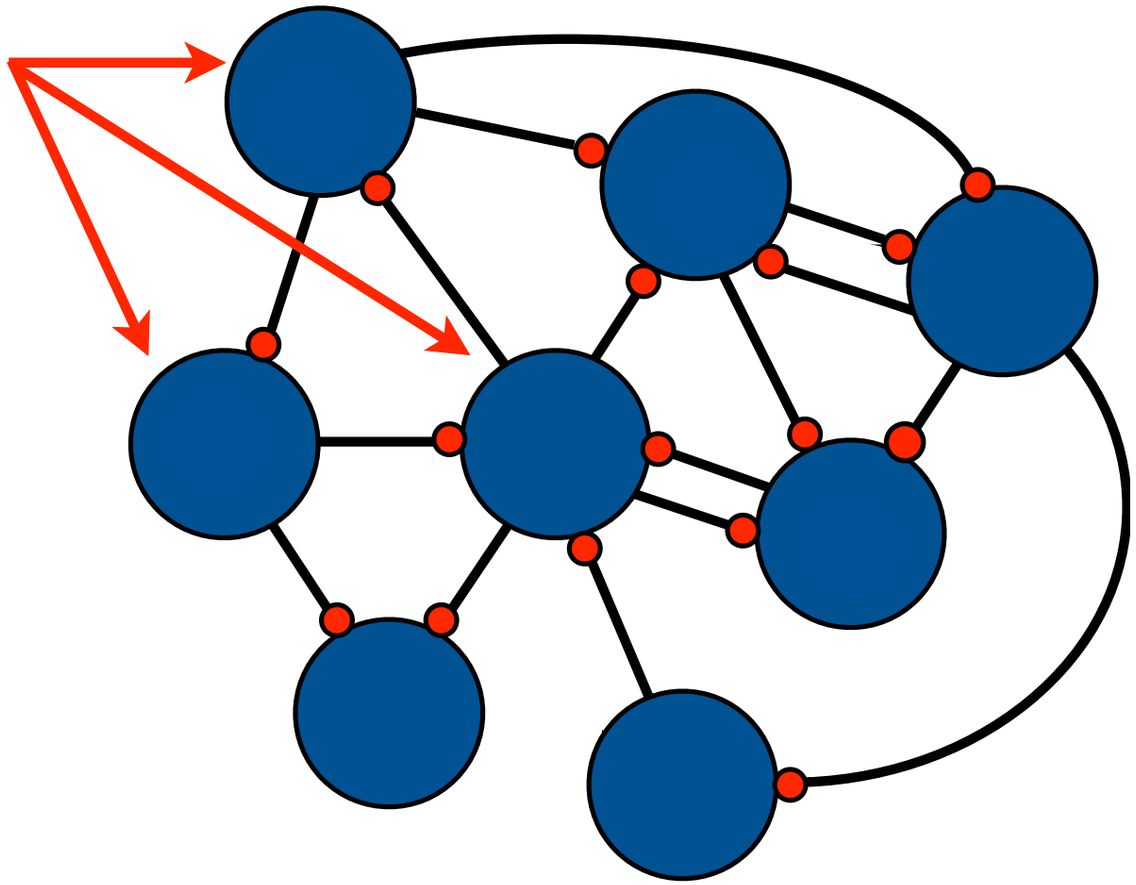


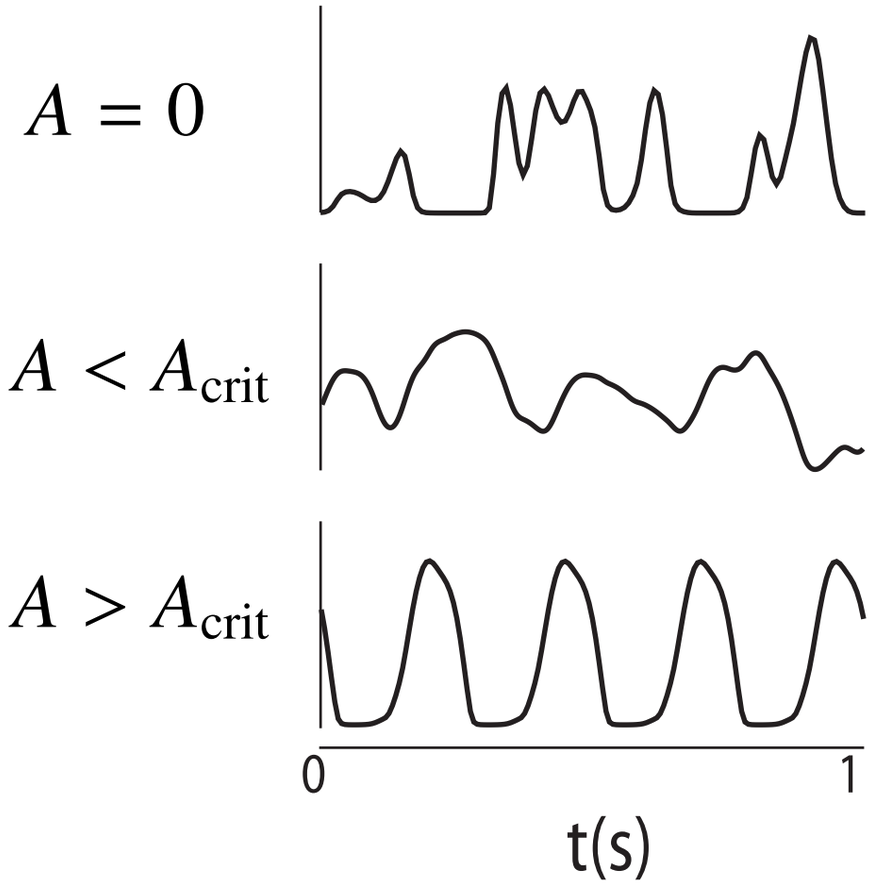


Sompolinsky, Crisanti & Sommers, 1988

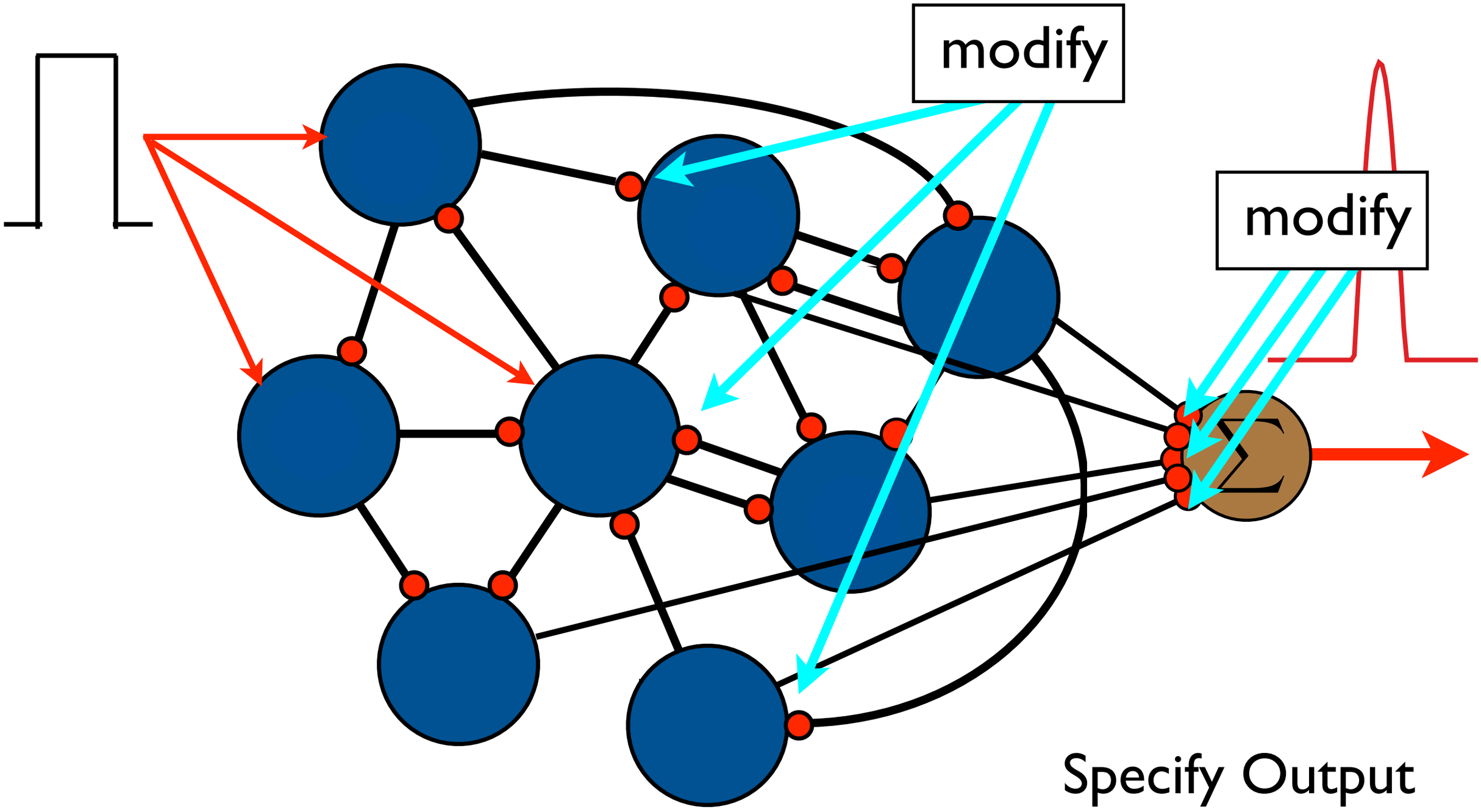

tanh(x)
0
t
1000

$$I(t) = A\cos(\omega t)$$



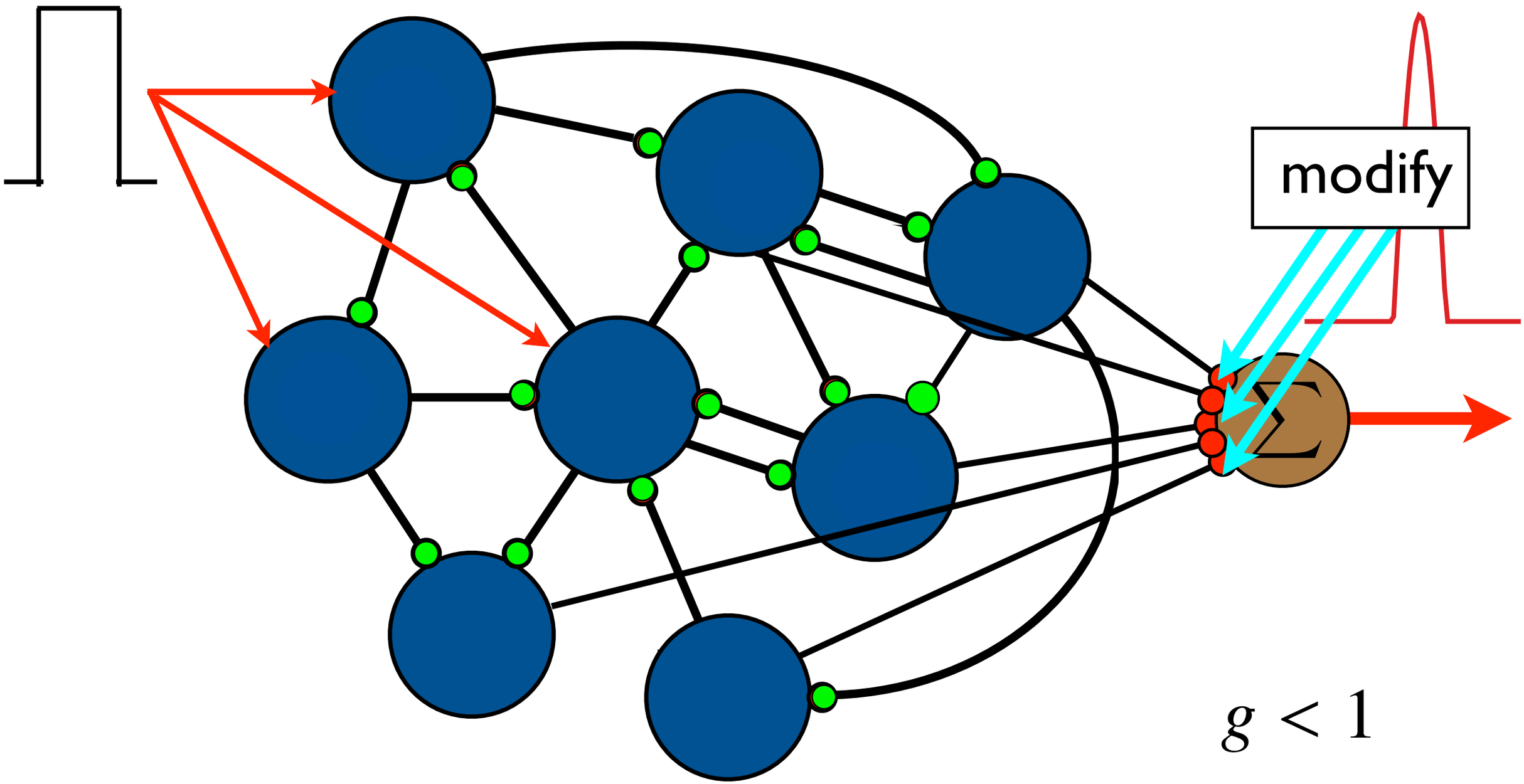


Rajan, Abbott & Sompolinsky, 2010



Specify Output

Backpropagation

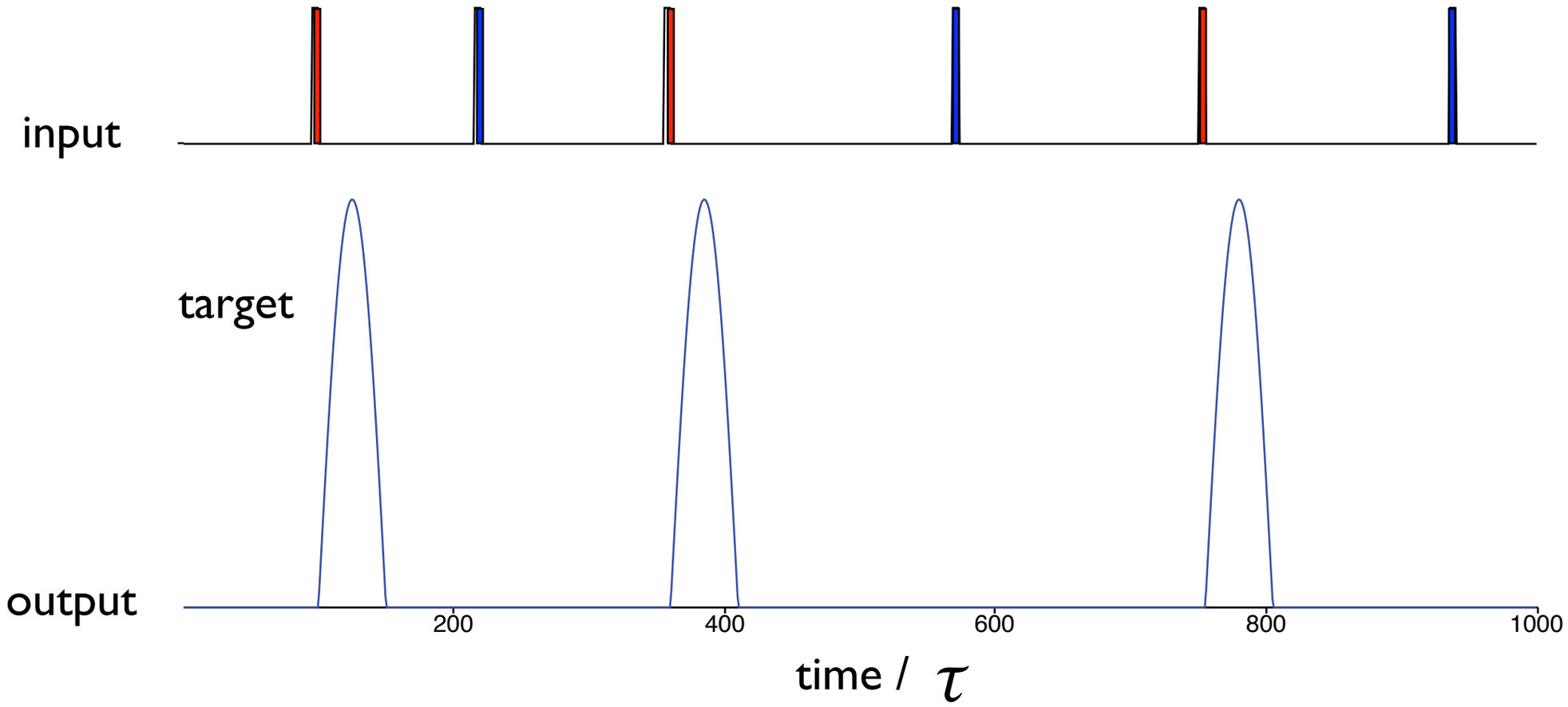


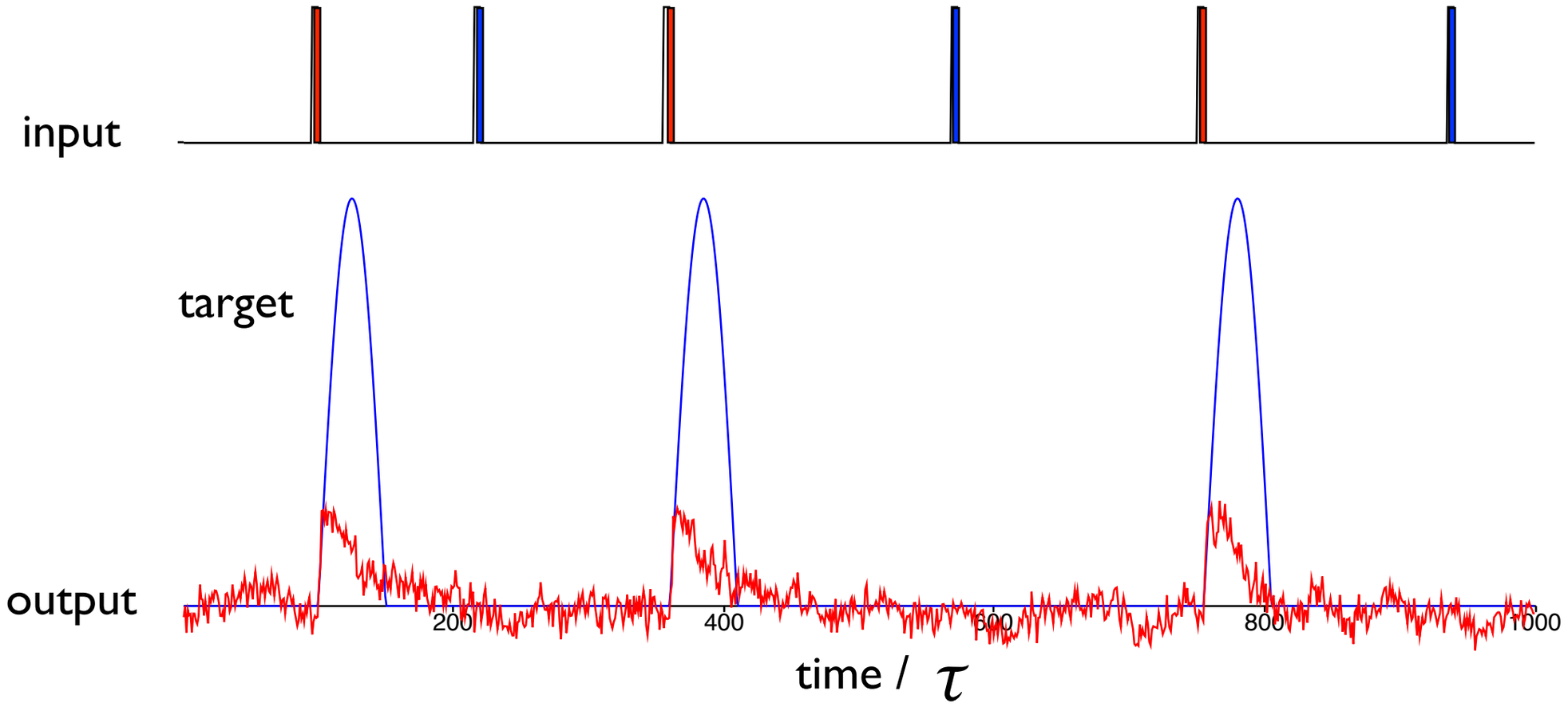
$g < 1$

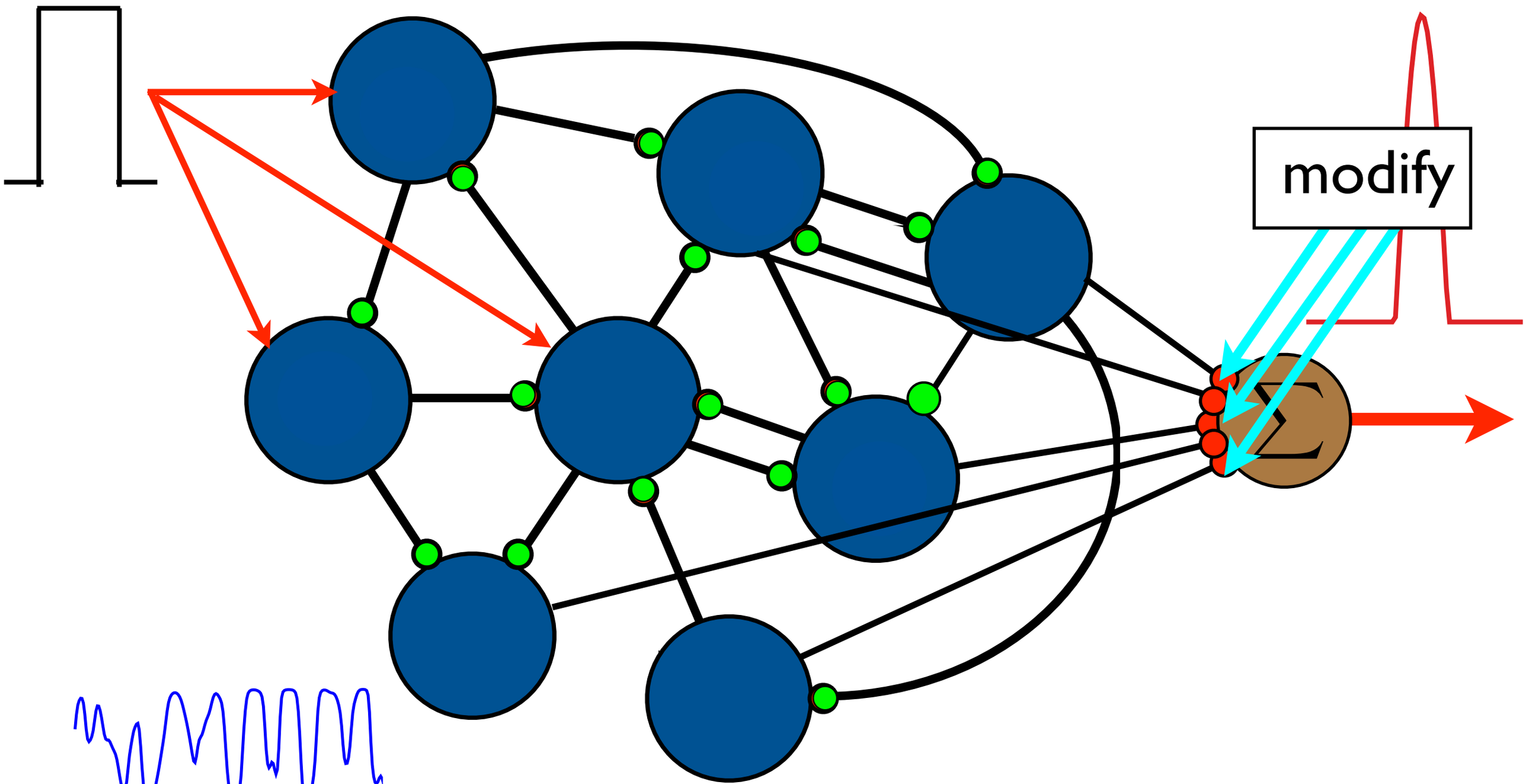
Specify Output

Liquid/Echo State

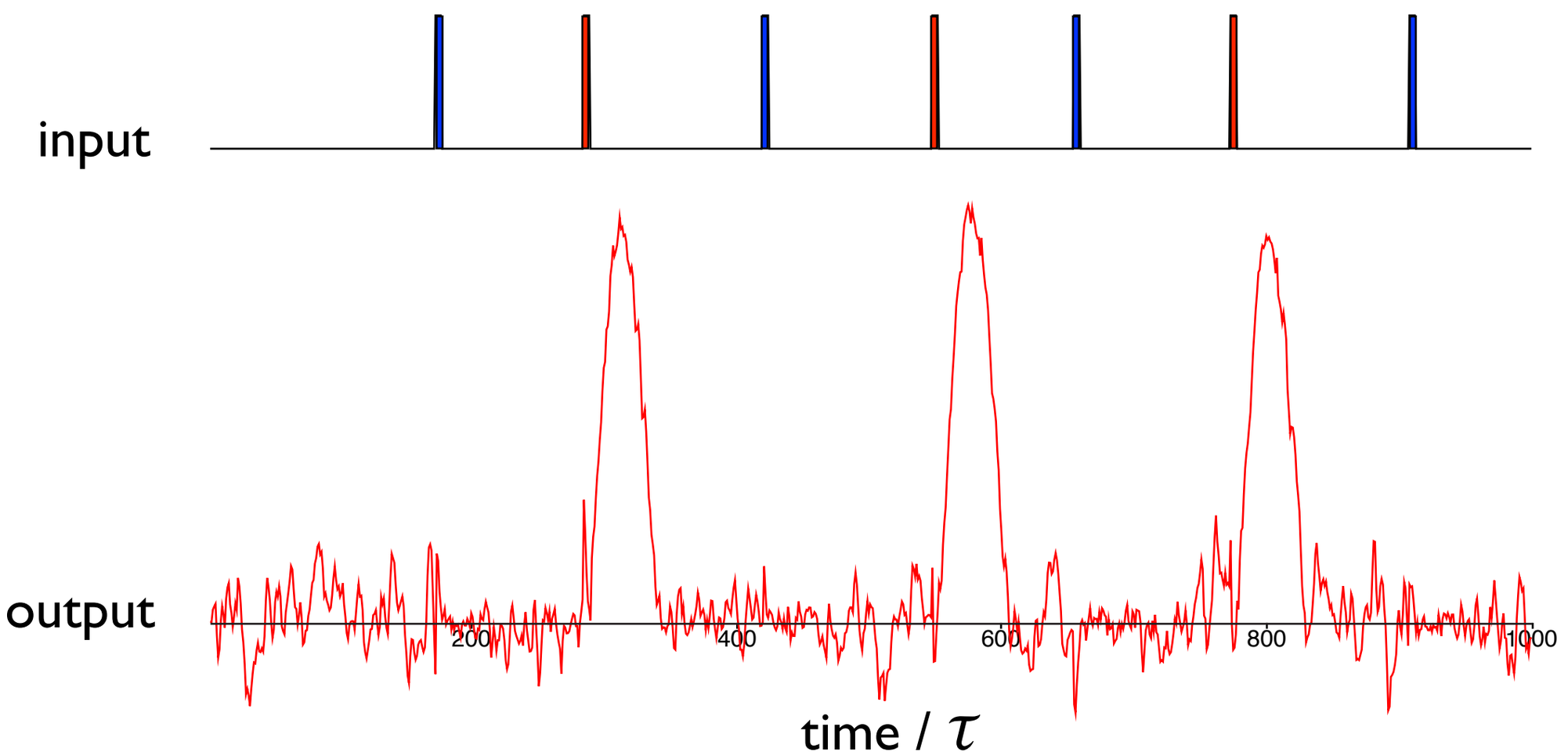
Jaeger & Maass

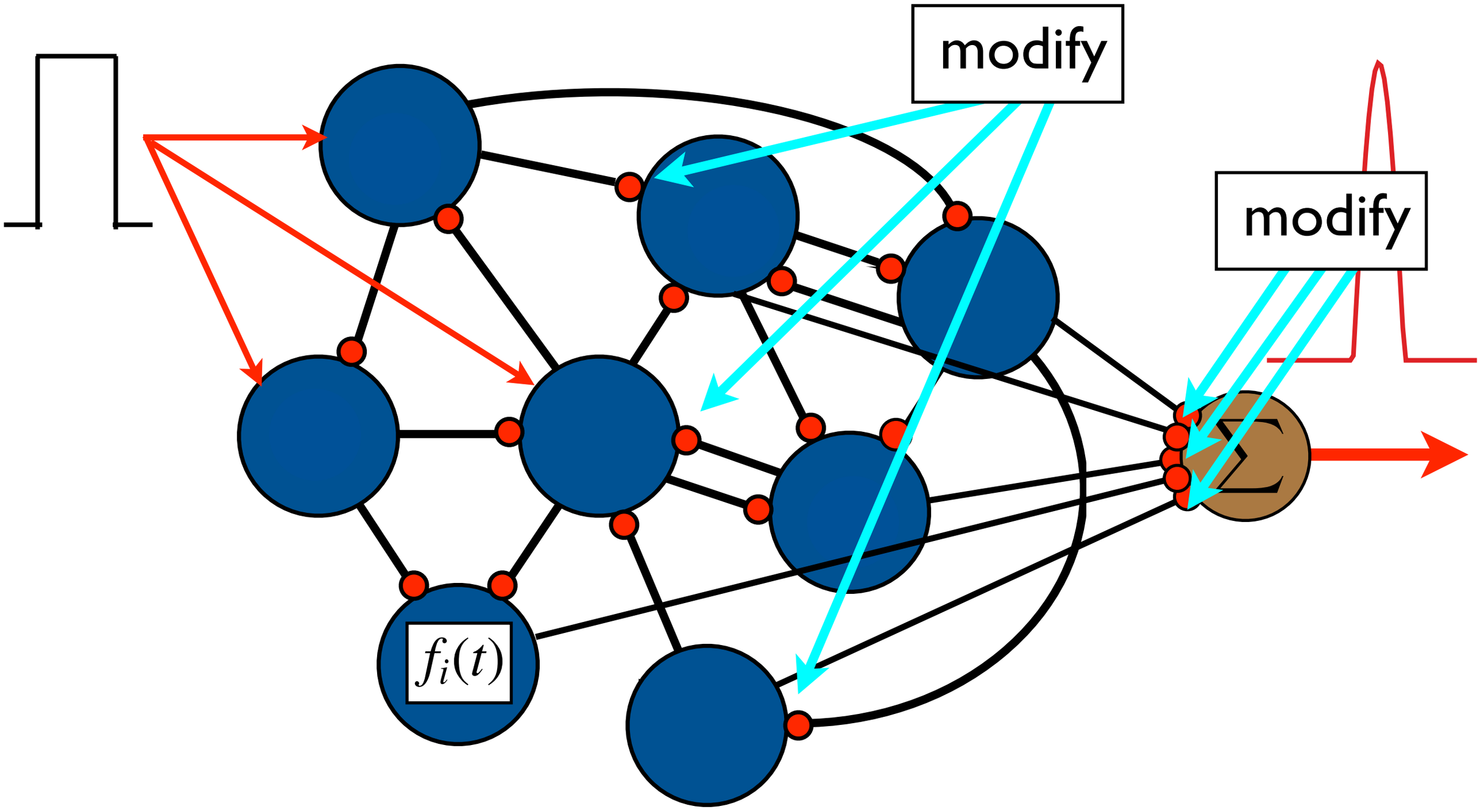

input
target
output
200
400
600
800
1000
time / τ


input
target
output
200
400
600
800
1000
time / τ



$g = 1.5$

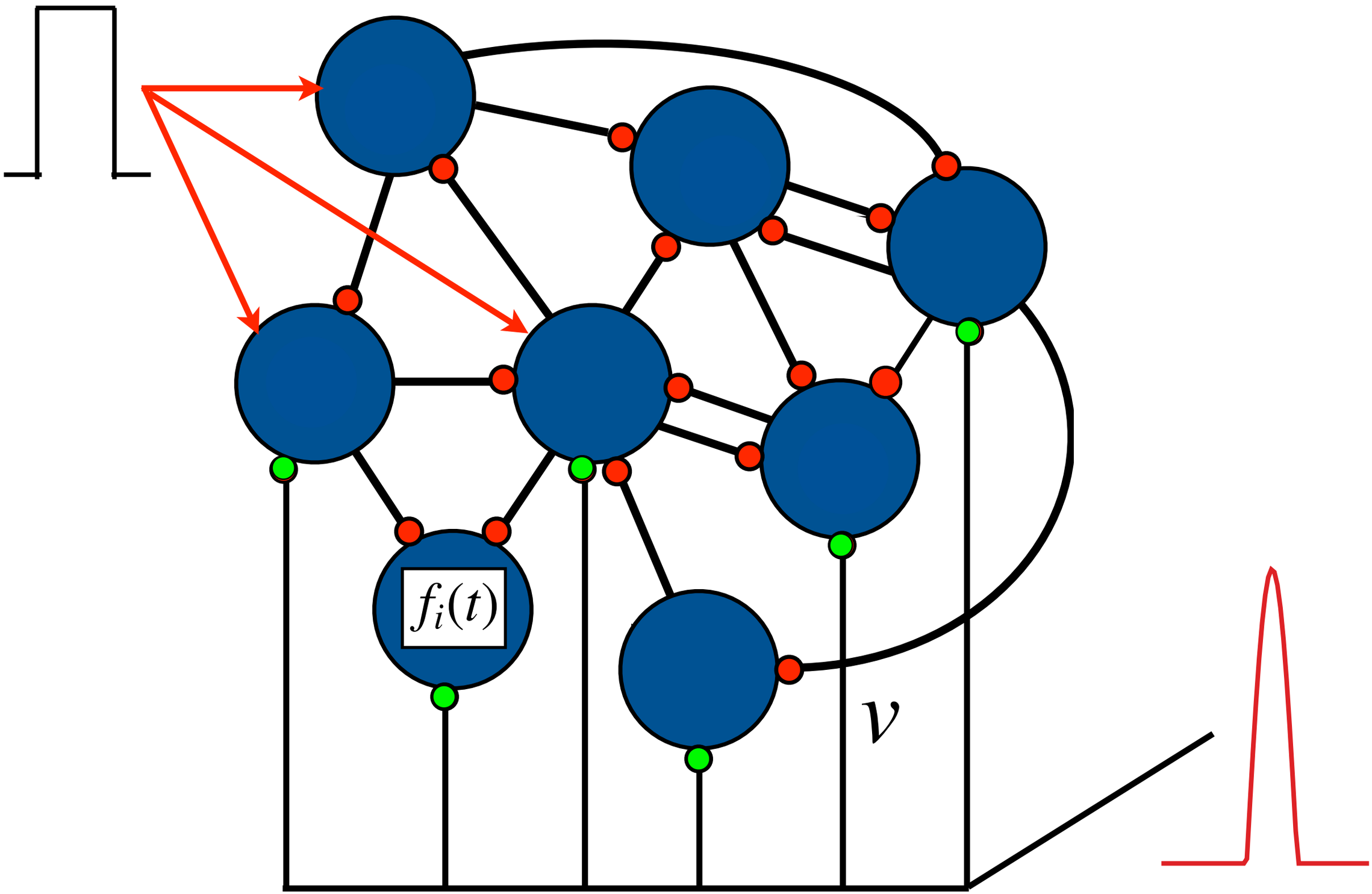

input
output
200
400
600
800
1000
time / τ



$$\left(\sum_{j=1}^{N} J_{ij} G(x_j) - f_i \right)^2$$

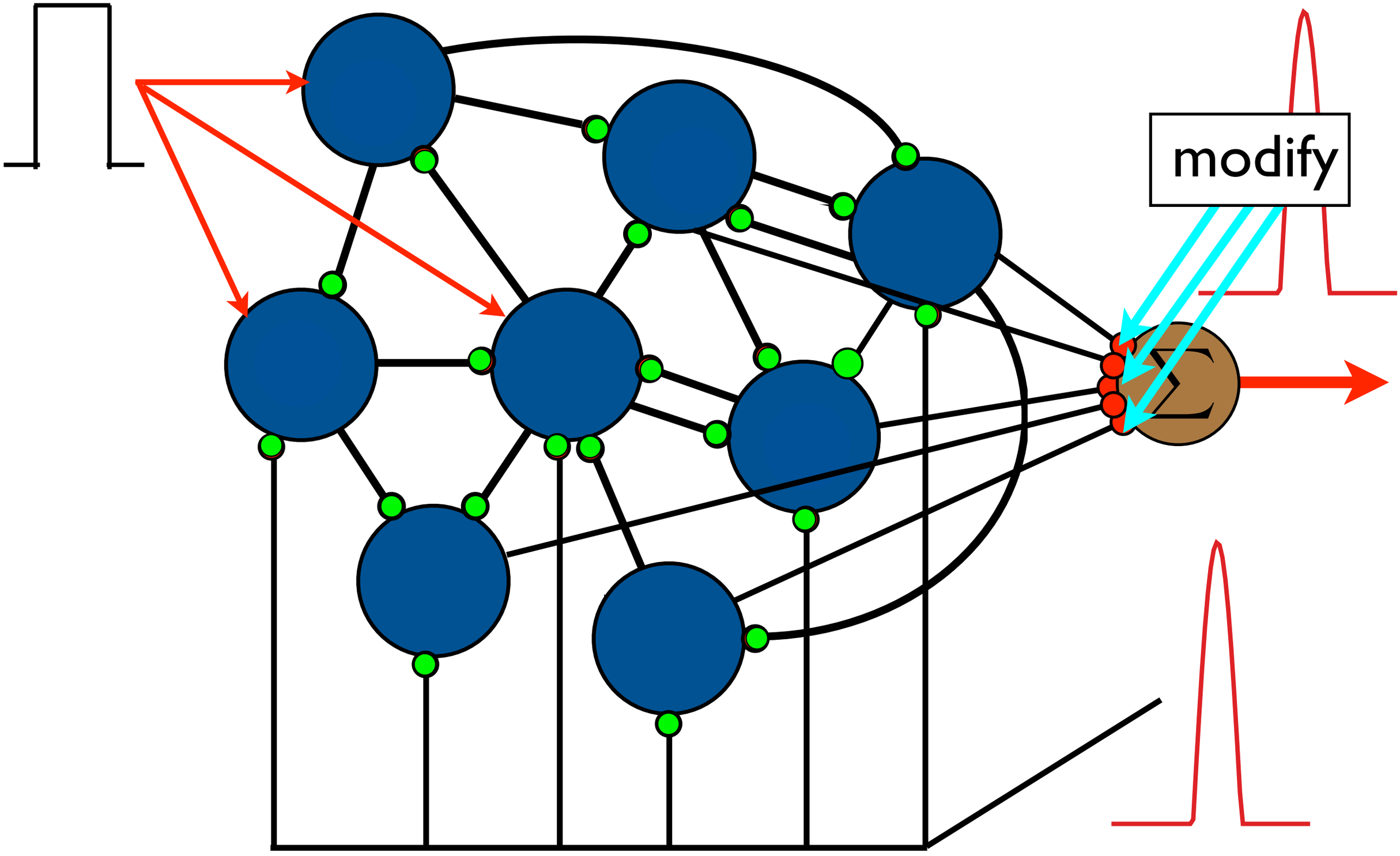
Specify Every Unit

Least Squares



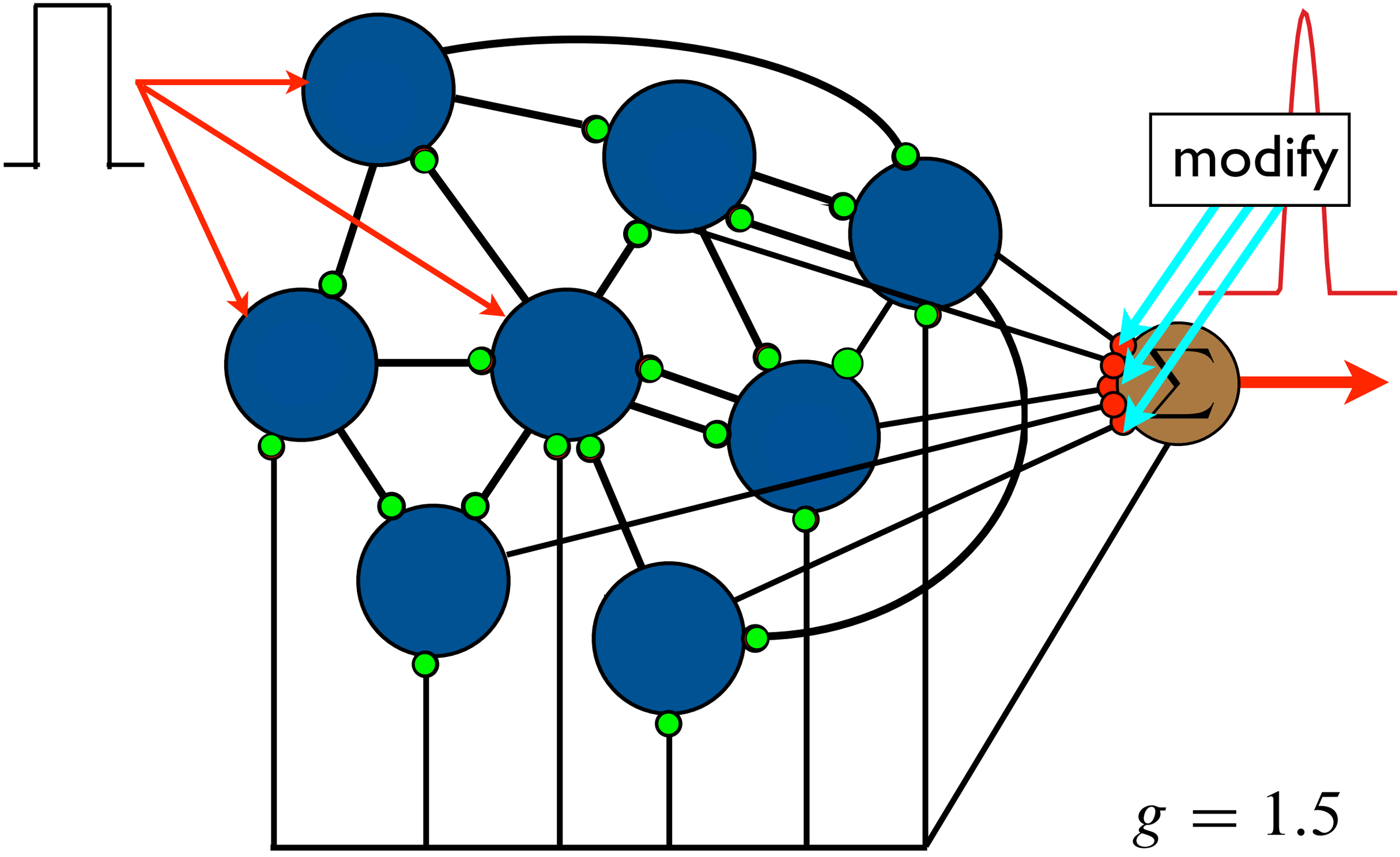
Target Propagation

Lee, Zhang, Biard, Bengio



Teacher Forcing

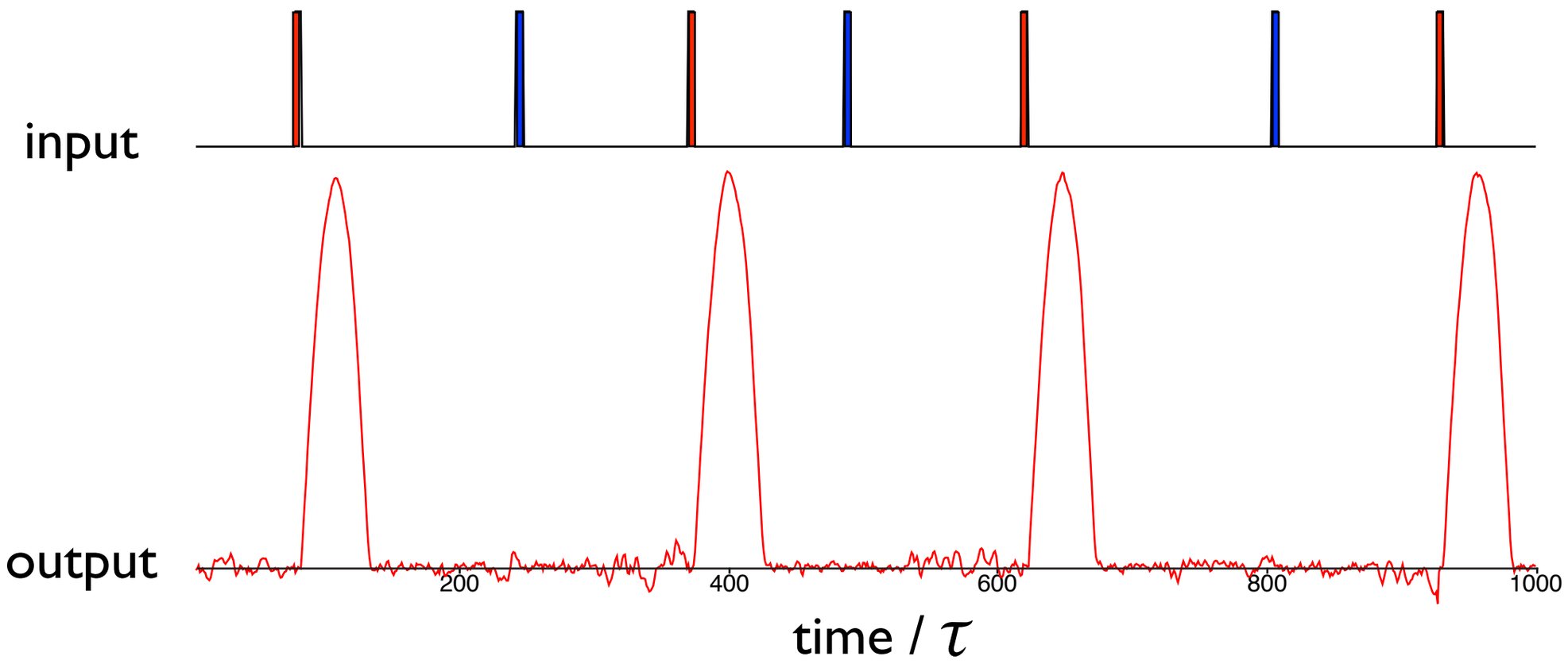
Jaeger, Haass

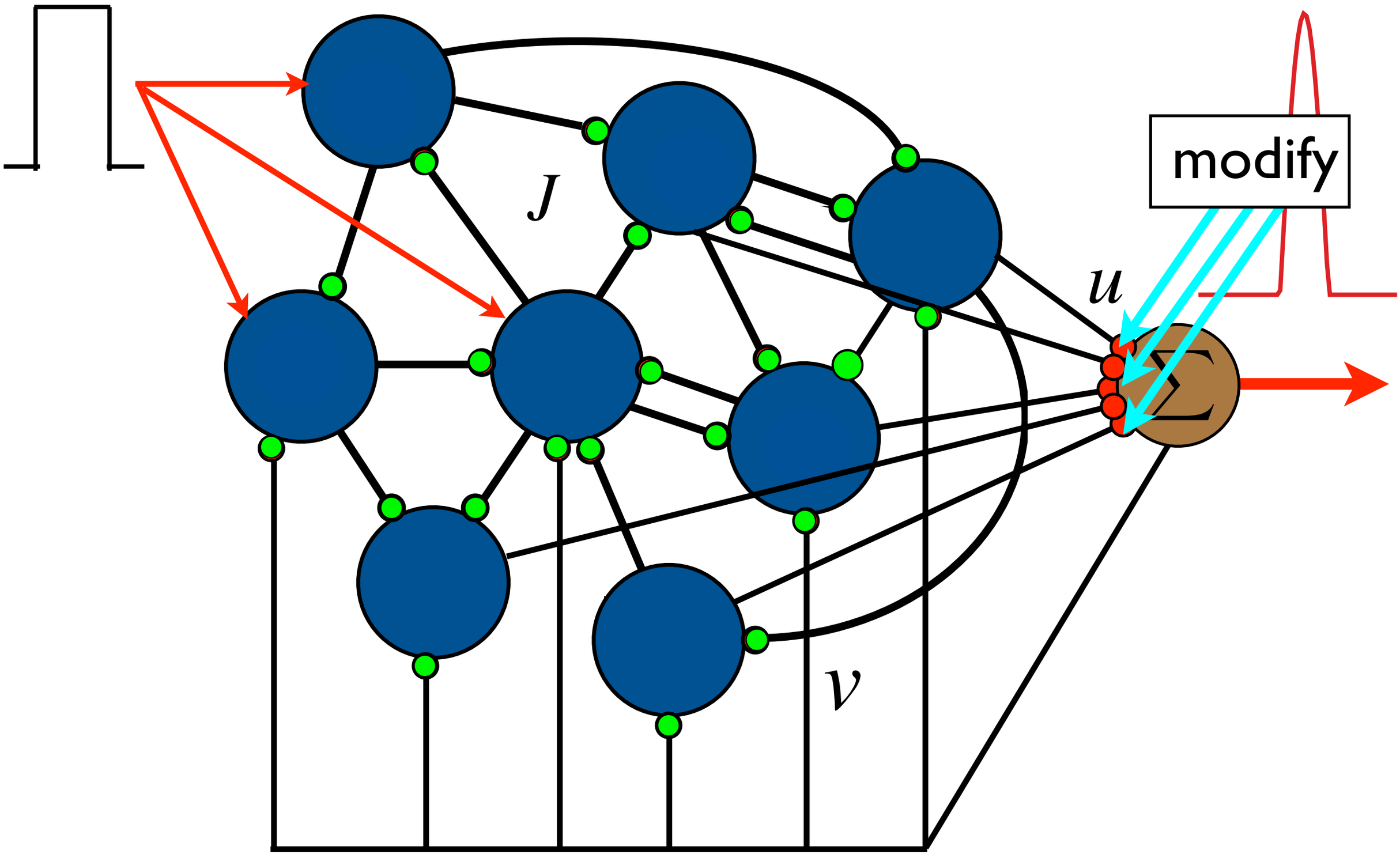


$g = 1.5$

FORCE

with David Sussillo


input
output
200
400
600
800
1000
time / τ


modify
J
u
v

$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} (J_{ij} + v_i u_j) \tanh(x_j)$$

adjust

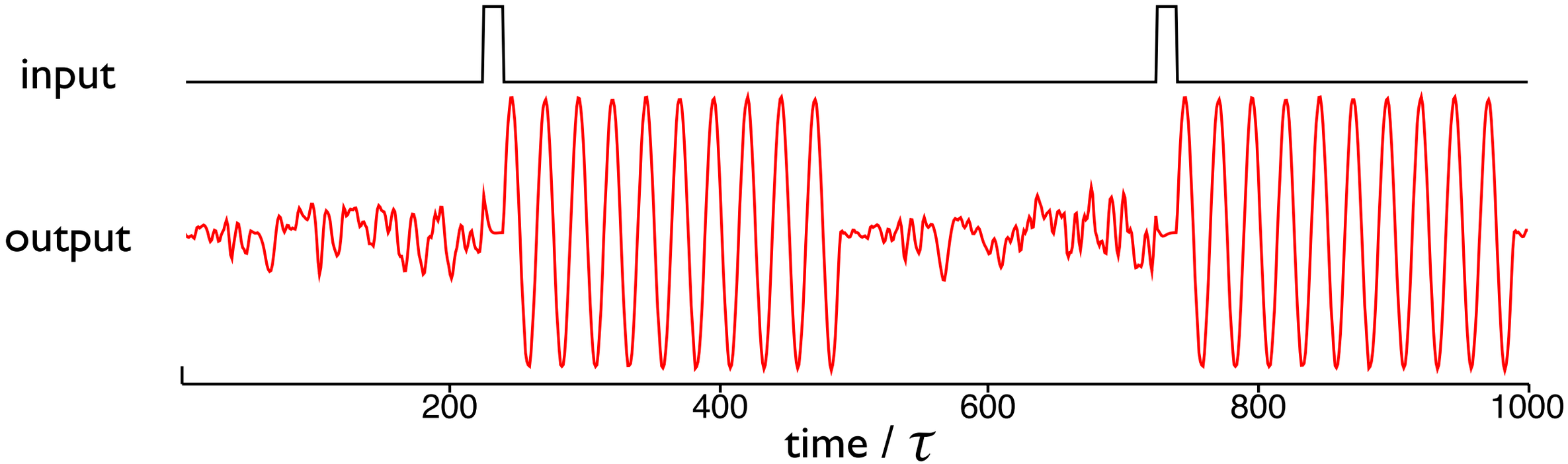
$$z(t) = \sum_{i=1}^{N} u_i \tanh(x_i) = f(t)$$

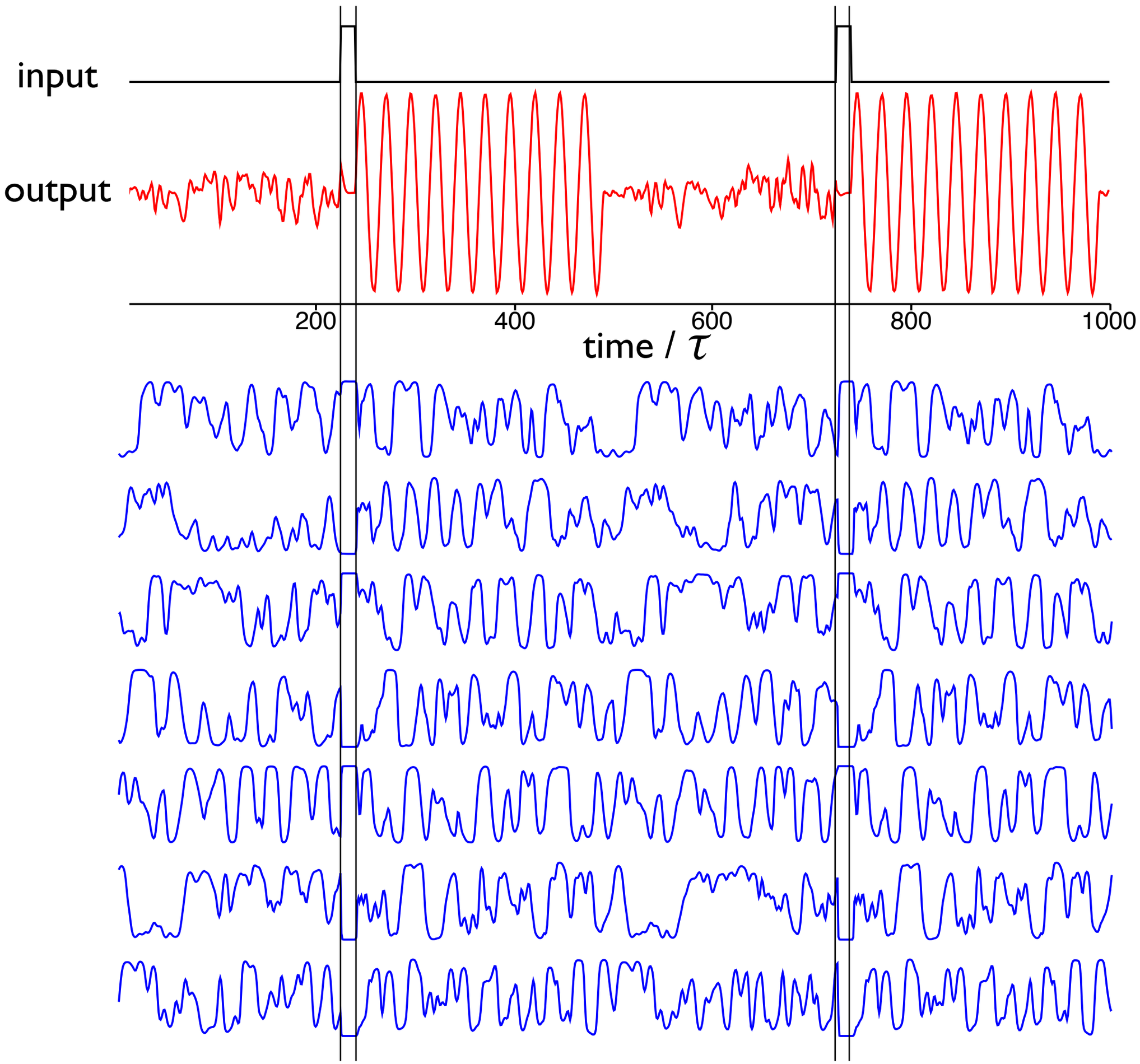
$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} (J_{ij} + v_i u_j) \tanh(x_j)$$

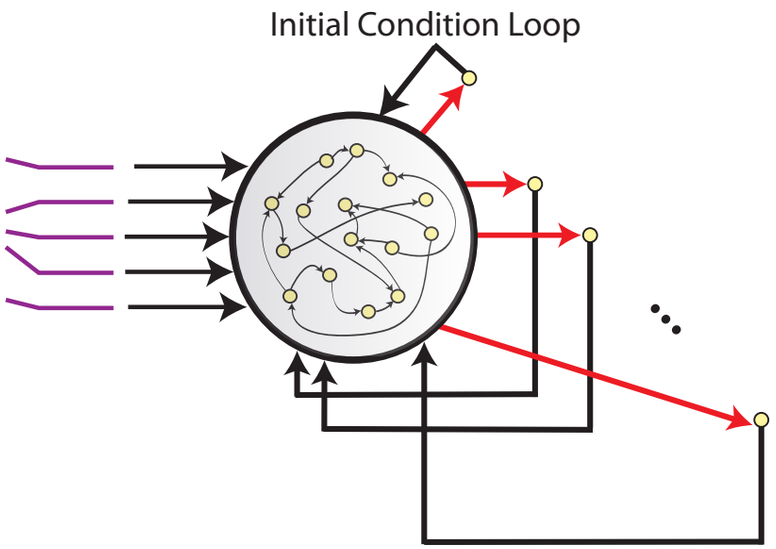
$$z(t) = \sum_{i=1}^{N} u_i \tanh(x_i) \rightarrow f(t)$$

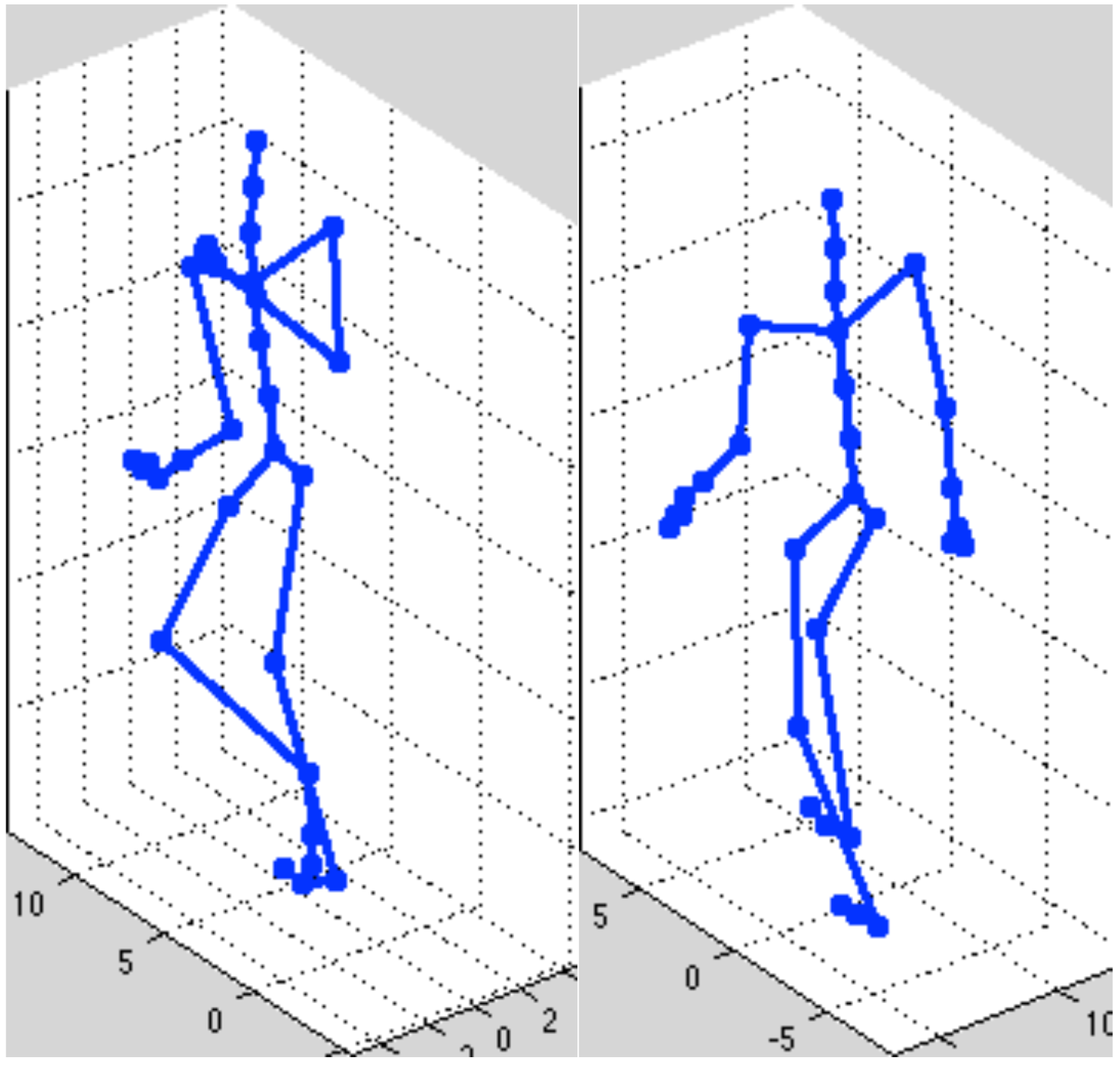
$$\Delta \vec{u} = (f - z) P \cdot \vec{r} \qquad \vec{r} = \tanh(\vec{x})$$

$$P_{\text{initial}} = \alpha I \qquad \Delta P = \frac{(P\vec{r})(\vec{r}^{\,T} P)}{1 + \vec{r}^{\,T} P \vec{r}}$$

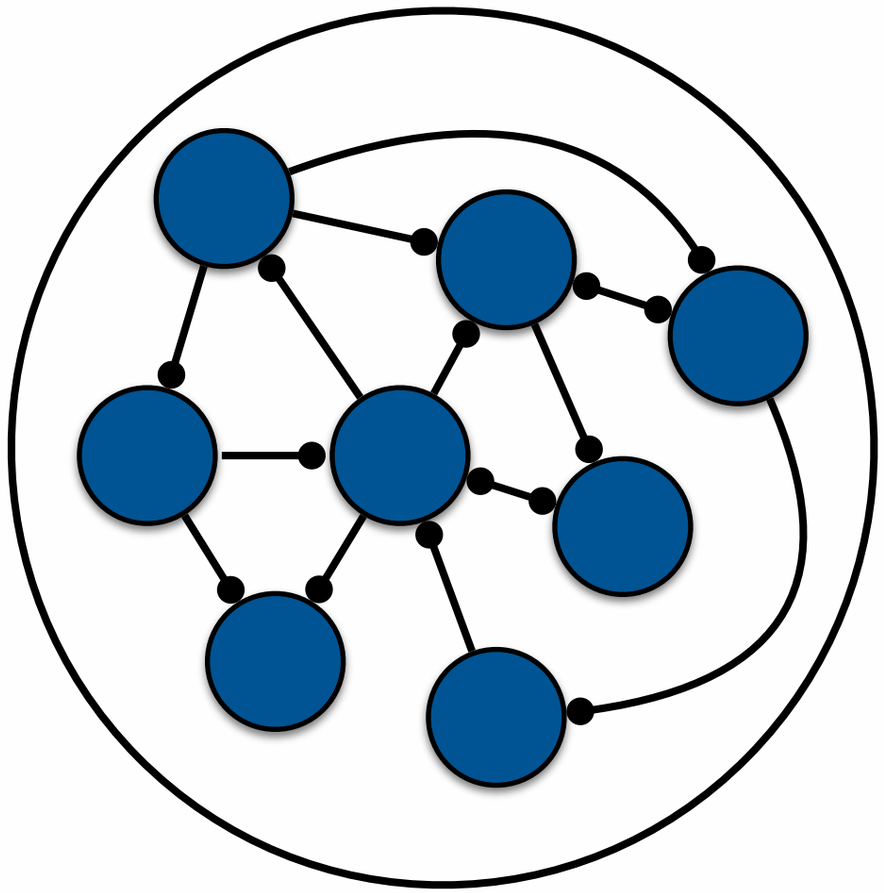

input
output
200
400
600
800
1000
time / τ


input
output
200
400
600
800
1000
time / τ

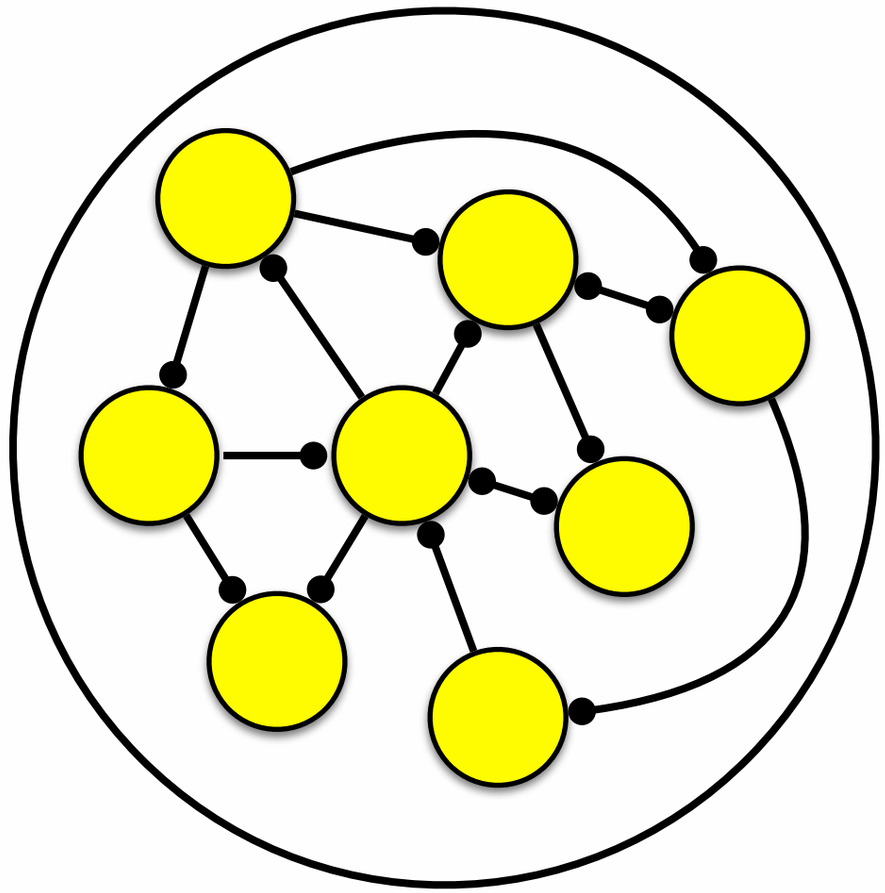

Initial Condition Loop


10
5
0
0
2
5
0
-5

firing rate model



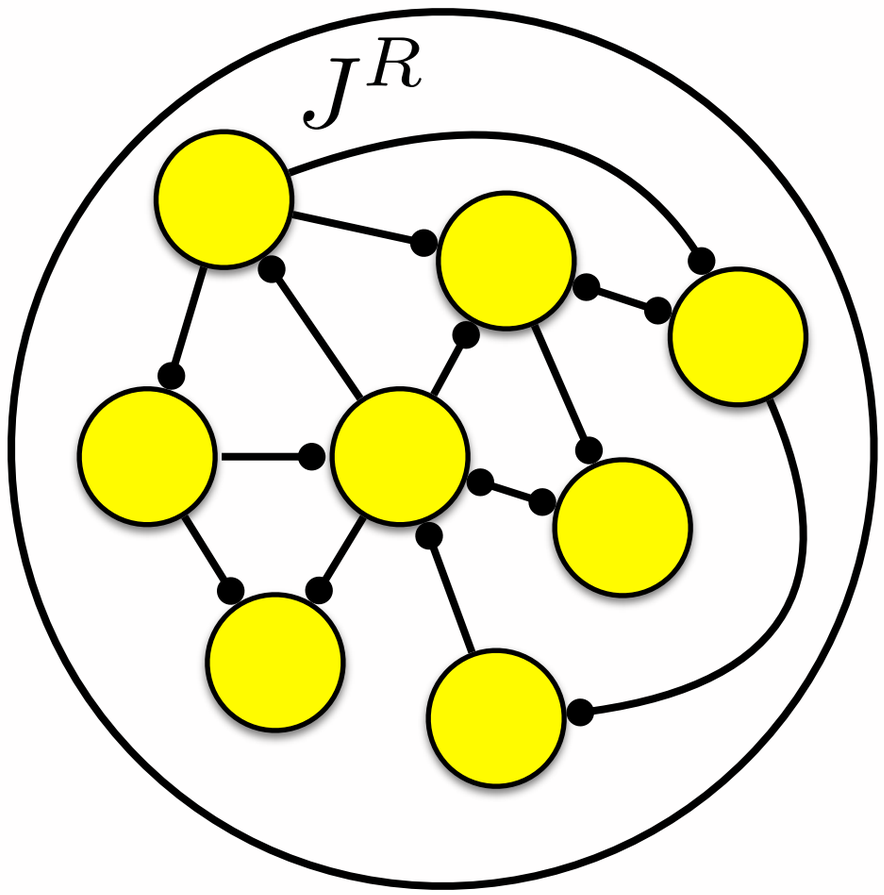
spiking model



?

with Brian DePasquale
and Mark Churchland

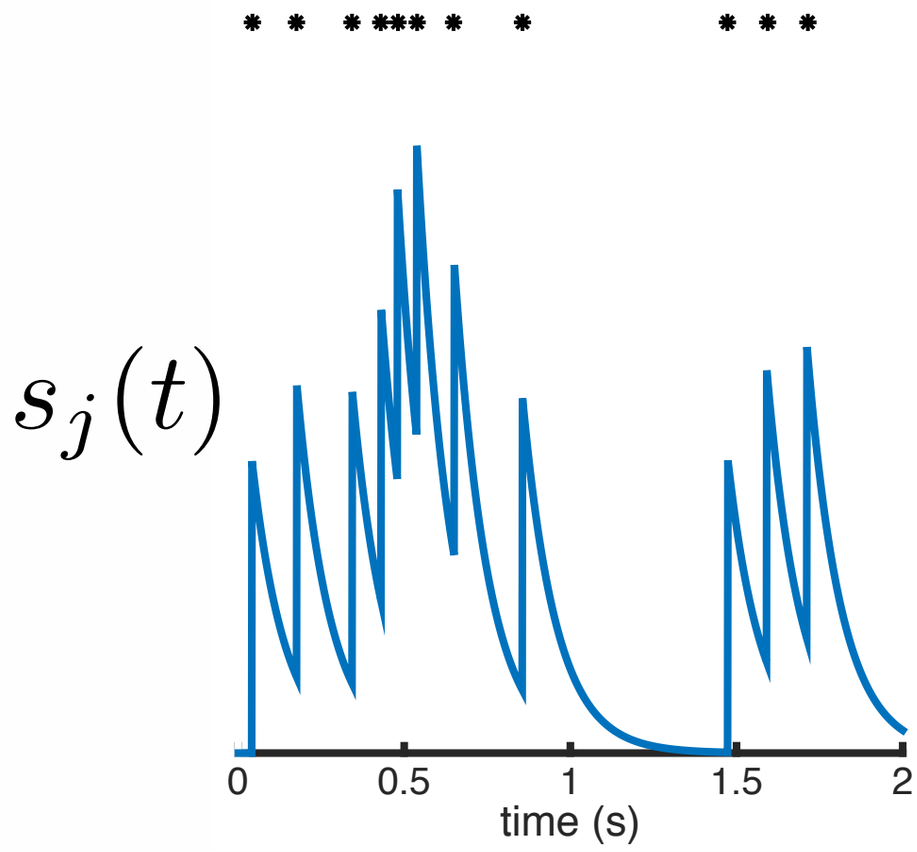
Eliasmith 2005, Boerlin, Machens, Deneve 2013
Thalmeier, Uhlmann, Kappen, Memmesheimer 2015

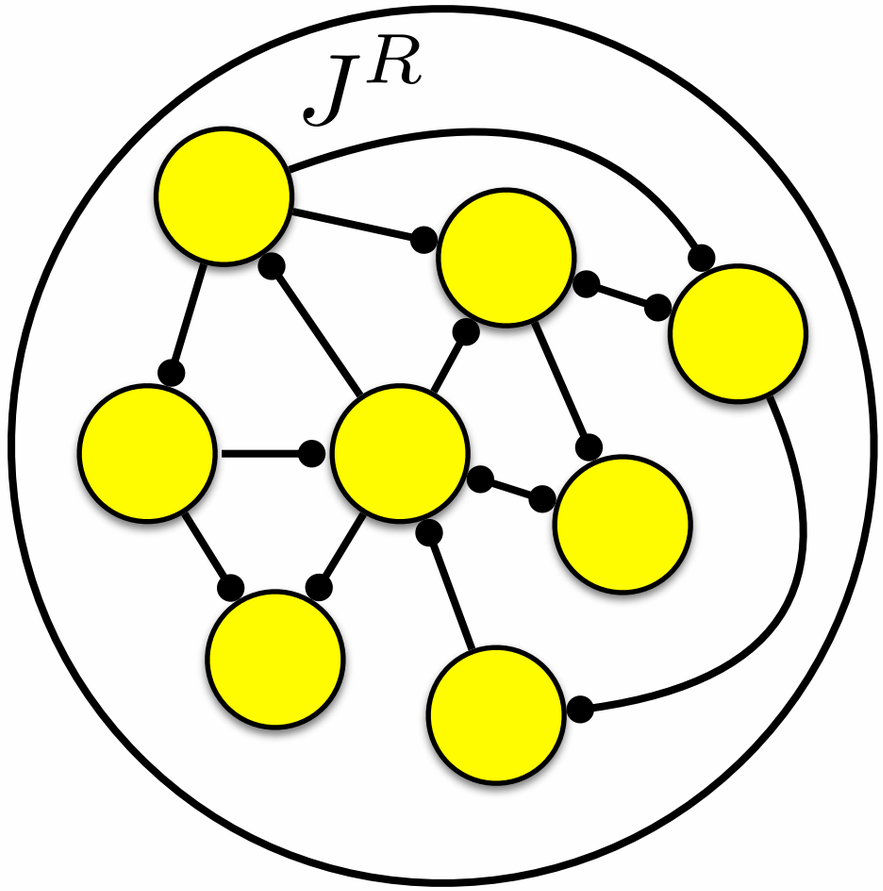


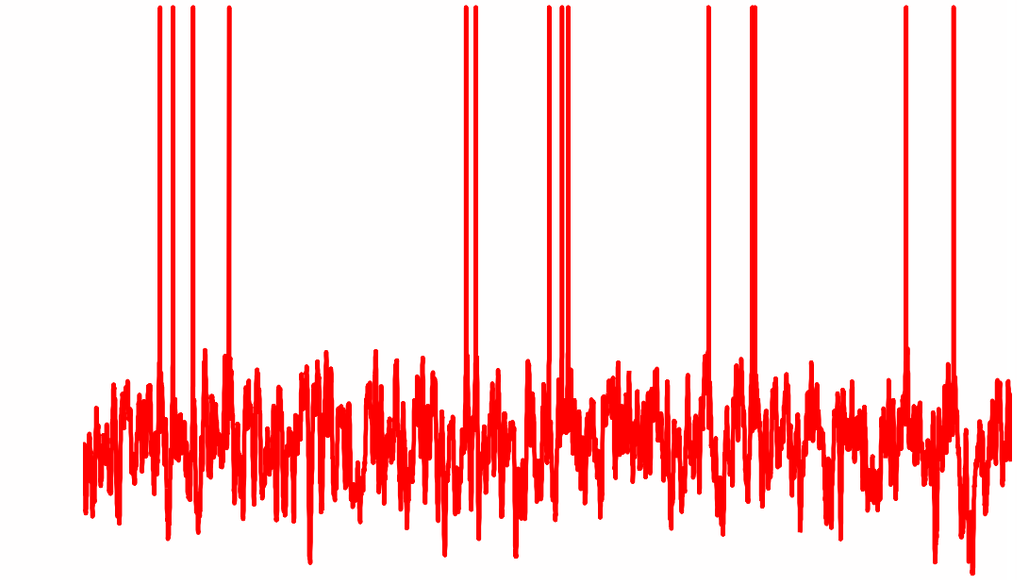
$$\tau_V \frac{dV_i}{dt} = V_{\text{rest}} - V_i + \sum_{j=1}^{N} J_{ij}^R s_j + w_i^{\text{in}} I$$

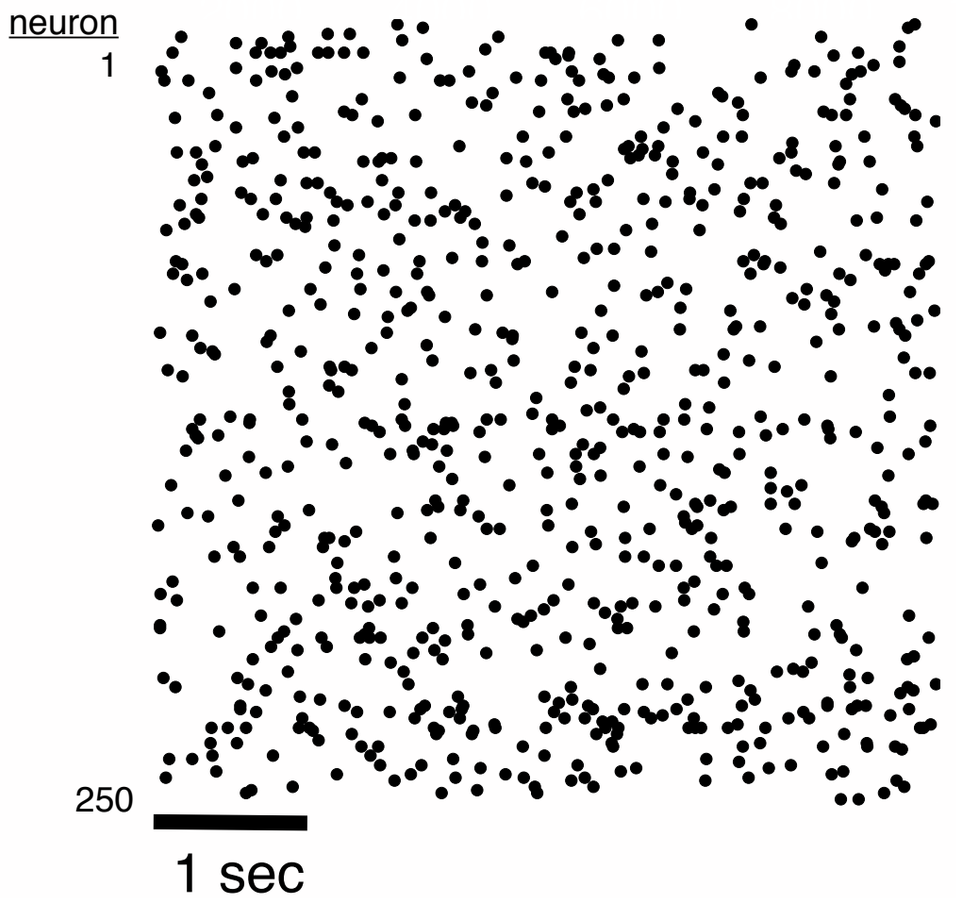
$$\langle J_{ij} \rangle = \mu \qquad \langle J_{ij}^2 \rangle = \sigma^2$$

$$\tau_s \frac{ds_i}{dt} = -s_i \qquad s \rightarrow s_i + 1$$

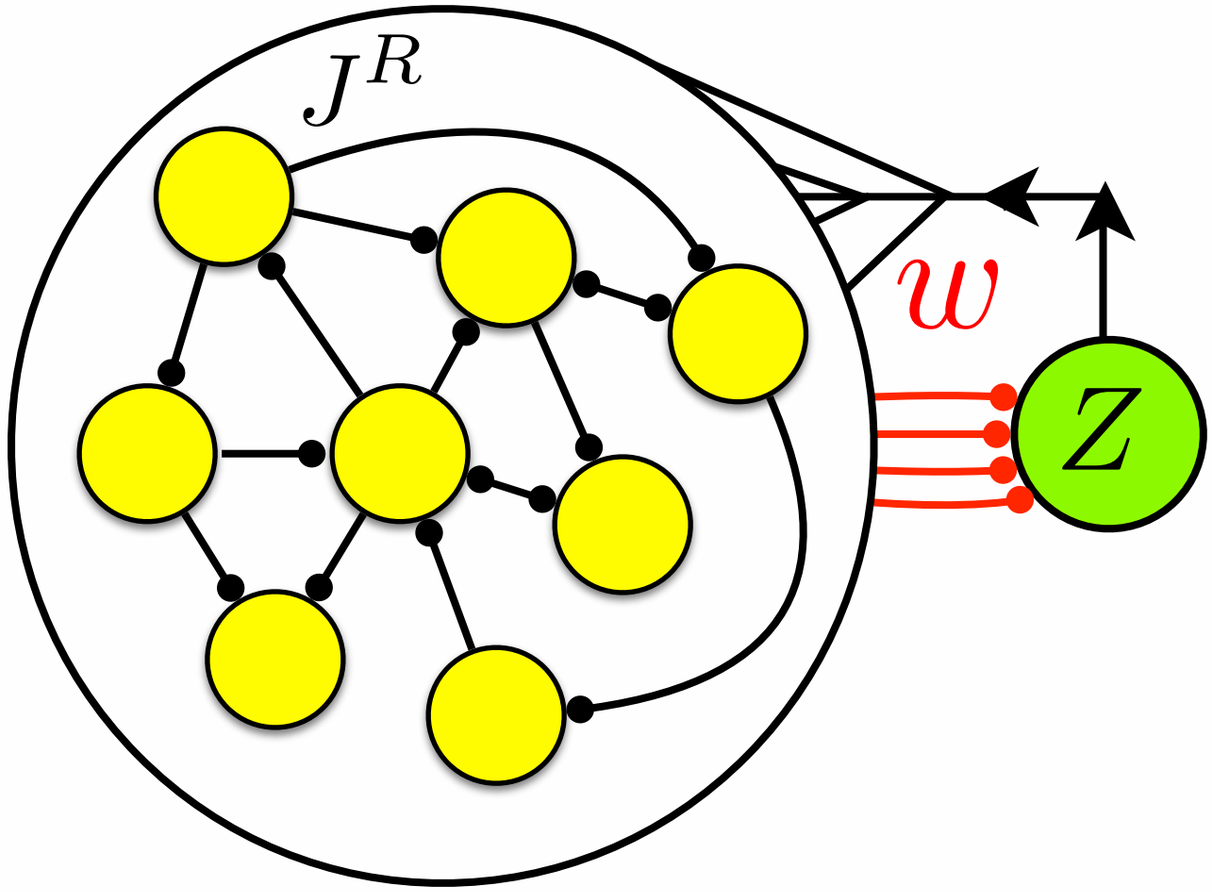


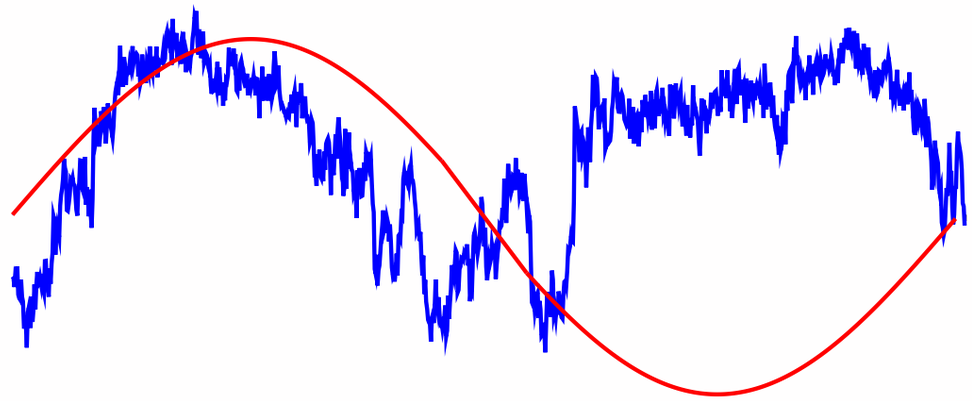


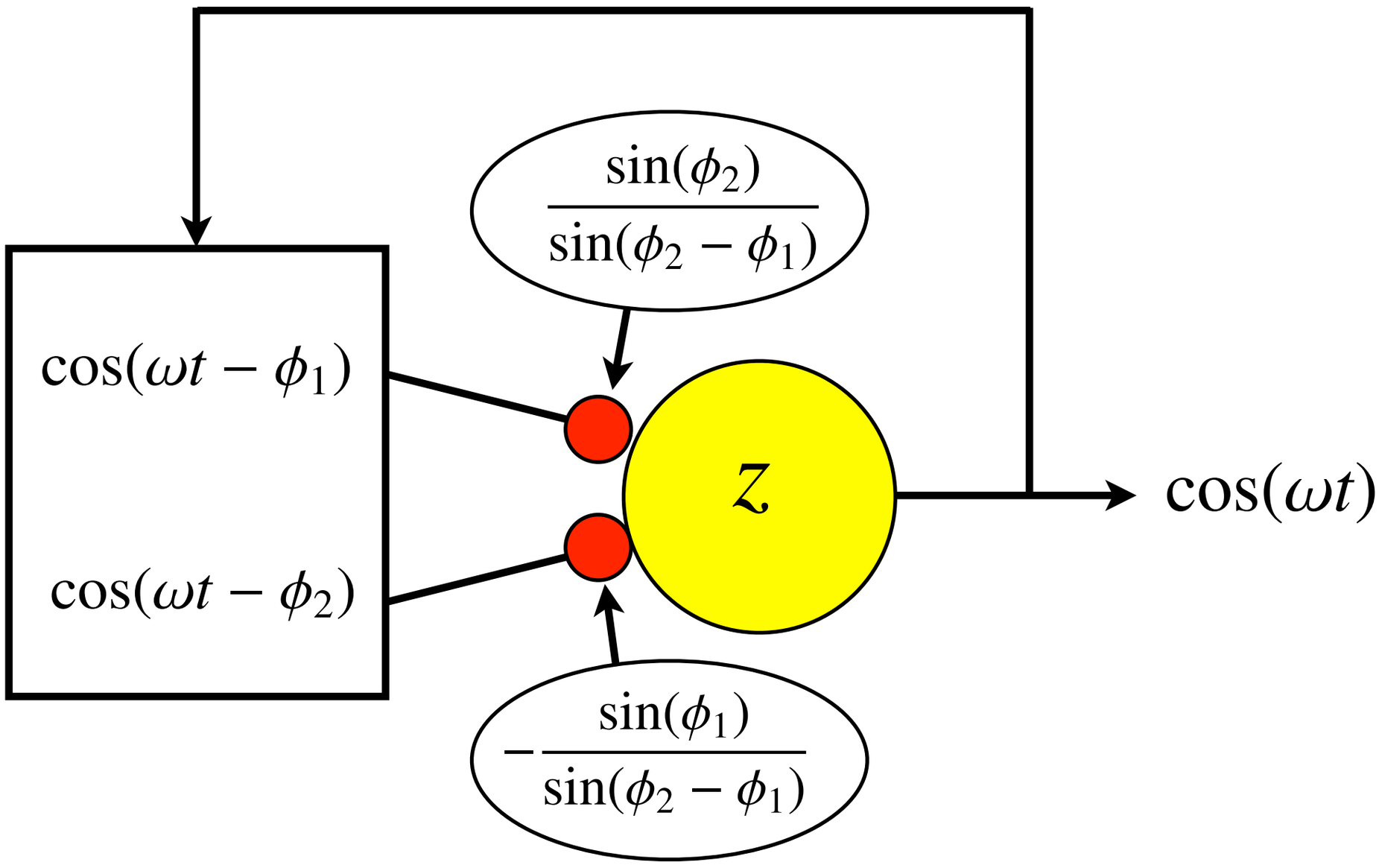




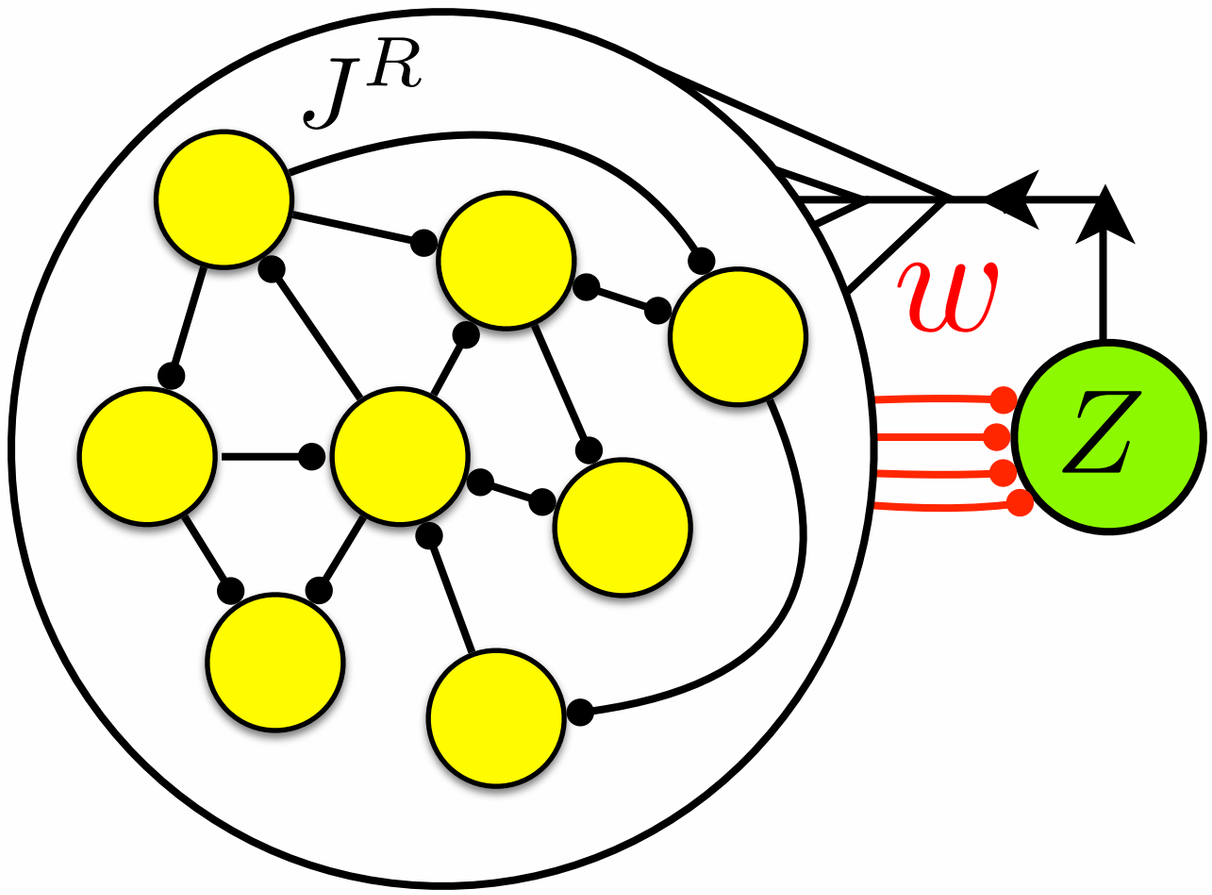
van Vreeswijk and Sompolinsky, 1996
Brunel, 2000

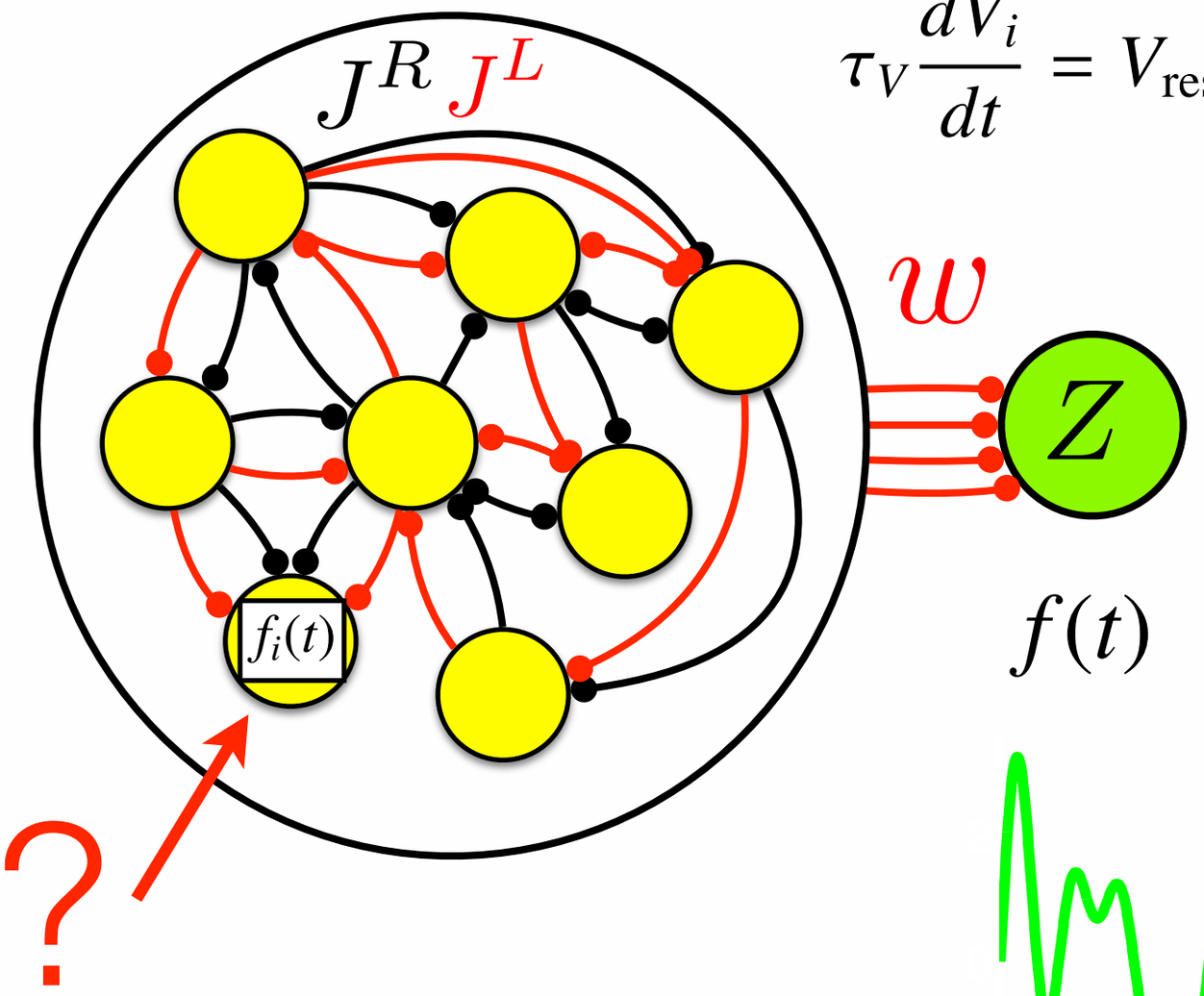

J^R
w
Z





$|\phi_1 - \phi_2|$ is small


J^R
w
Z



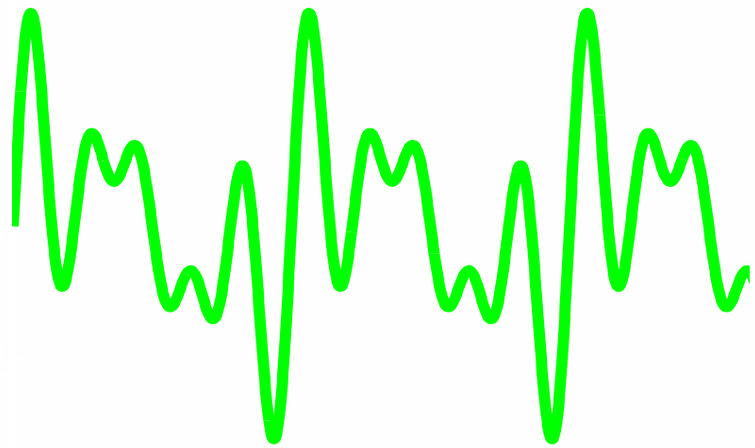
$$\tau_V \frac{dV_i}{dt} = V_{\text{rest}} - V_i + \sum_{j=1}^{N} J_{ij}^R s_j + \sum_{j=1}^{N} J_{ij}^L s_j + w_i^{\text{in}} I$$

w

Z

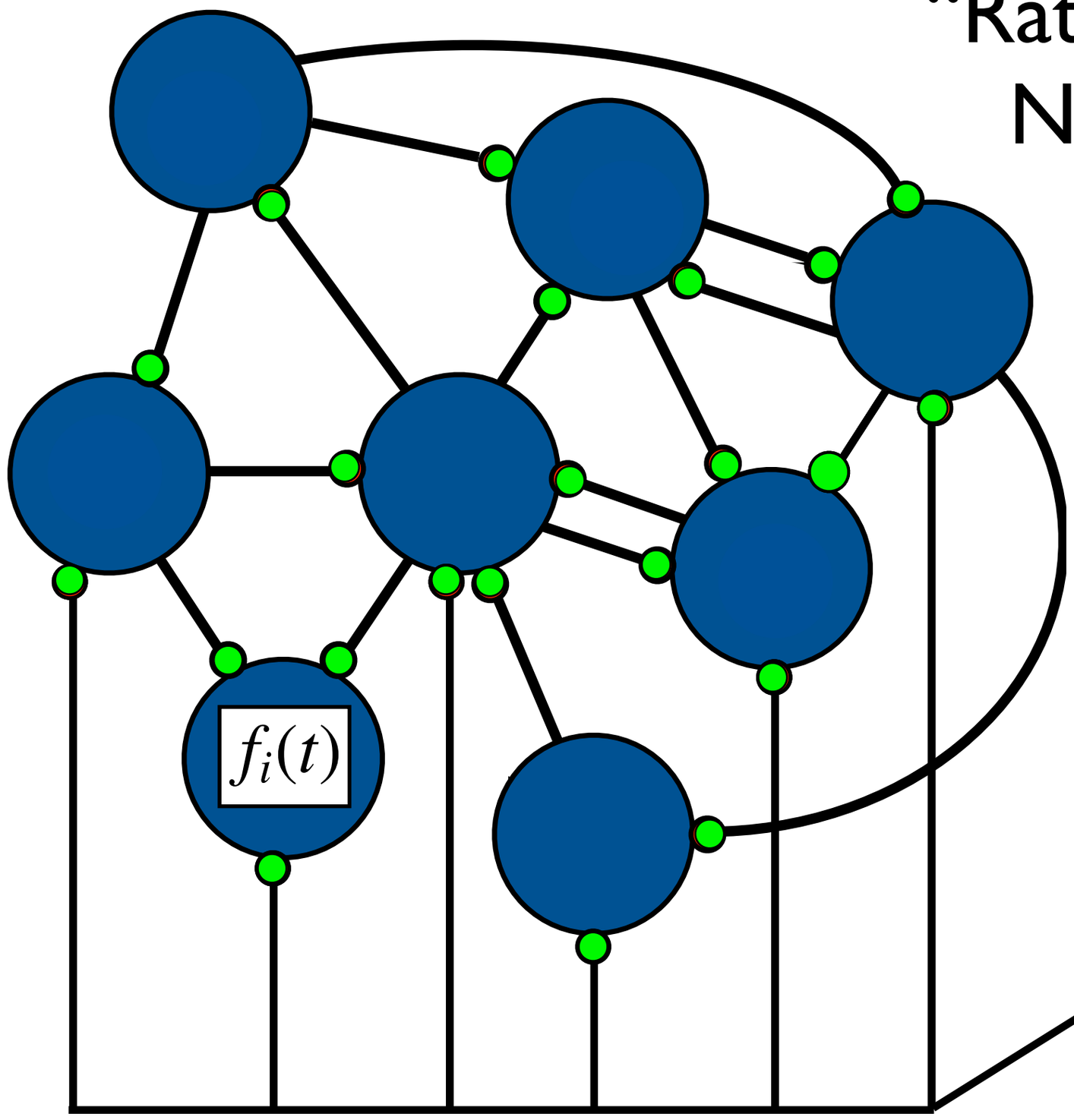
$f(t)$

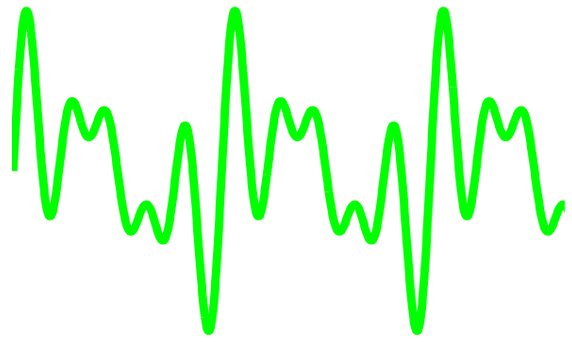
$$Z = \sum_{j=1}^{N} w_j s_j$$



$$E = \int_0^T dt \, ||J^L s(t) - f(t)||^2 \quad E = \int_0^T dt \, ||w s(t) - F(t)||^2$$

“Rate” Network!
No Learning





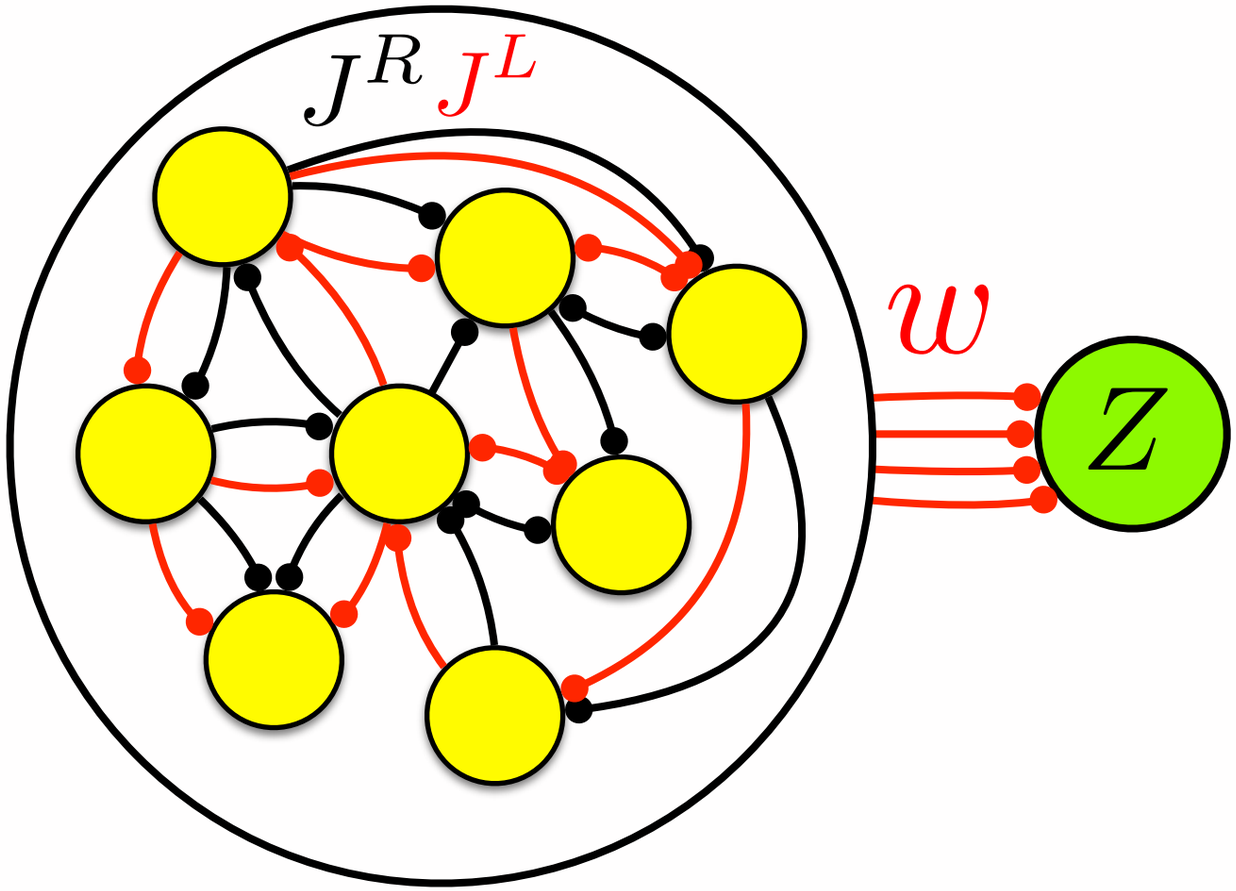
“rate”

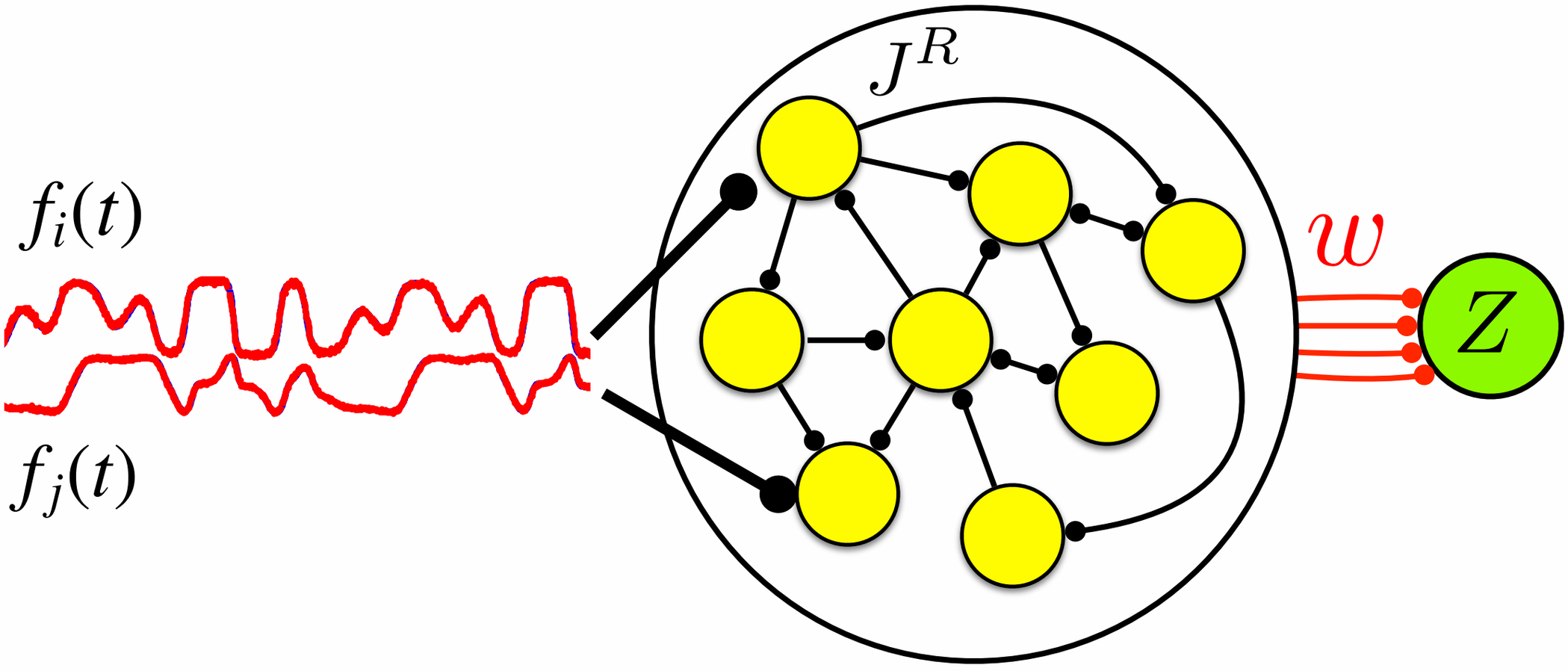
spiking

modify

modify

$f_i(t)$


J^R
J^L
w
Z



$$s(t)$$

$$E = \int_0^T ||J^L s(t) - f(t)||^2$$

$$E = \int_0^T ||w s(t) - F(t)||^2$$

spikes

$J^L s(t)$

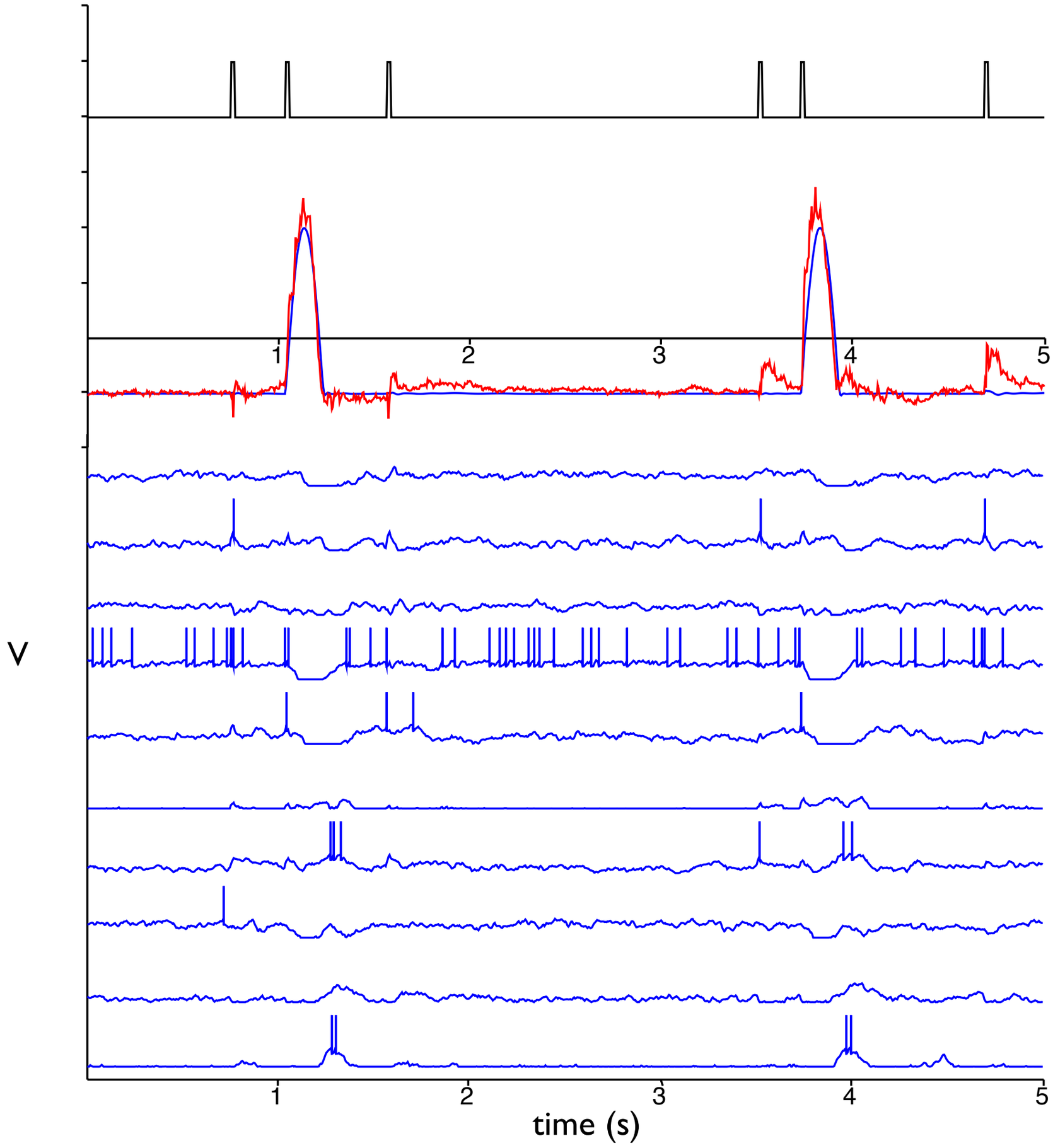
$Z(t)$

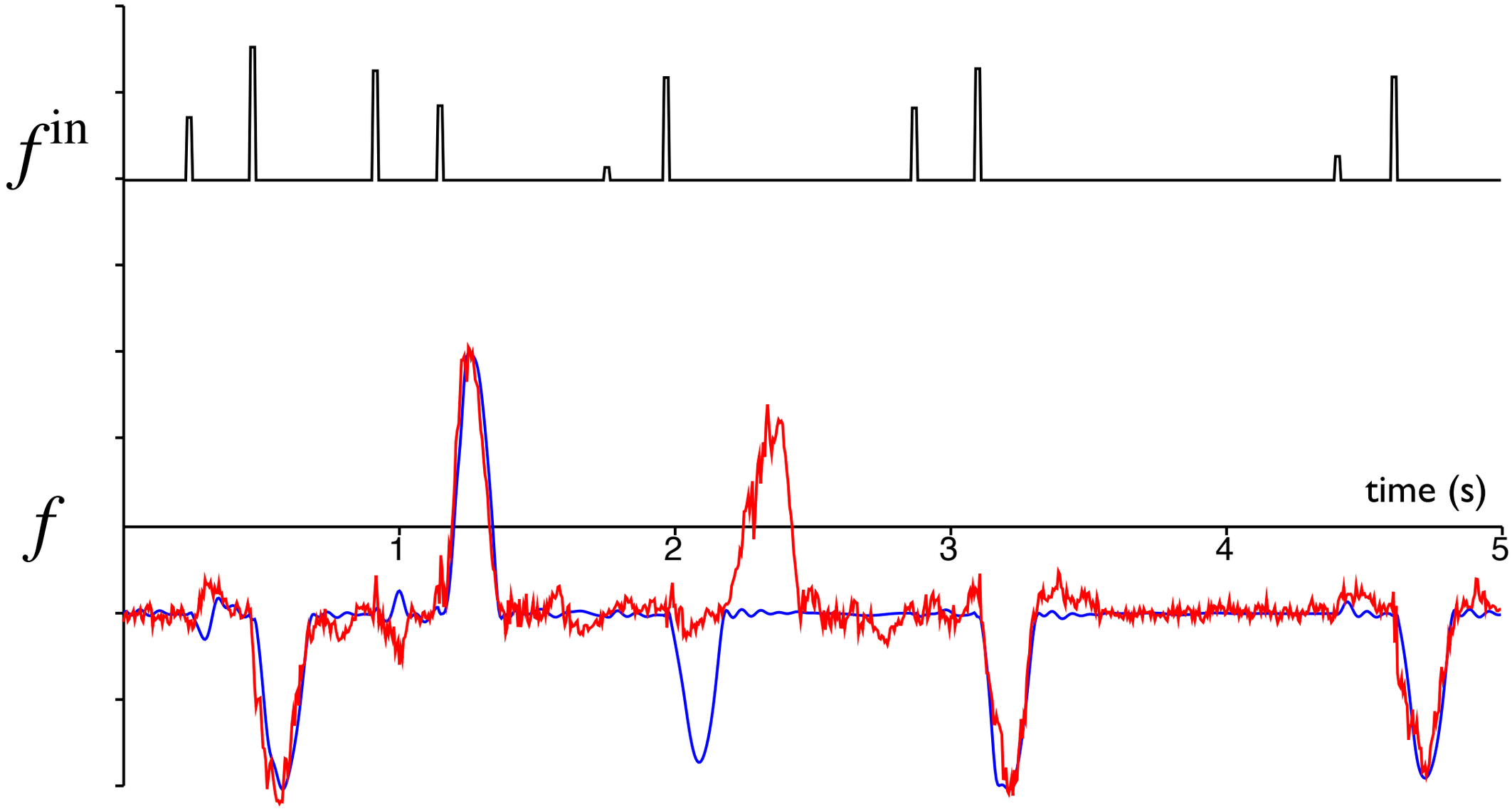
0

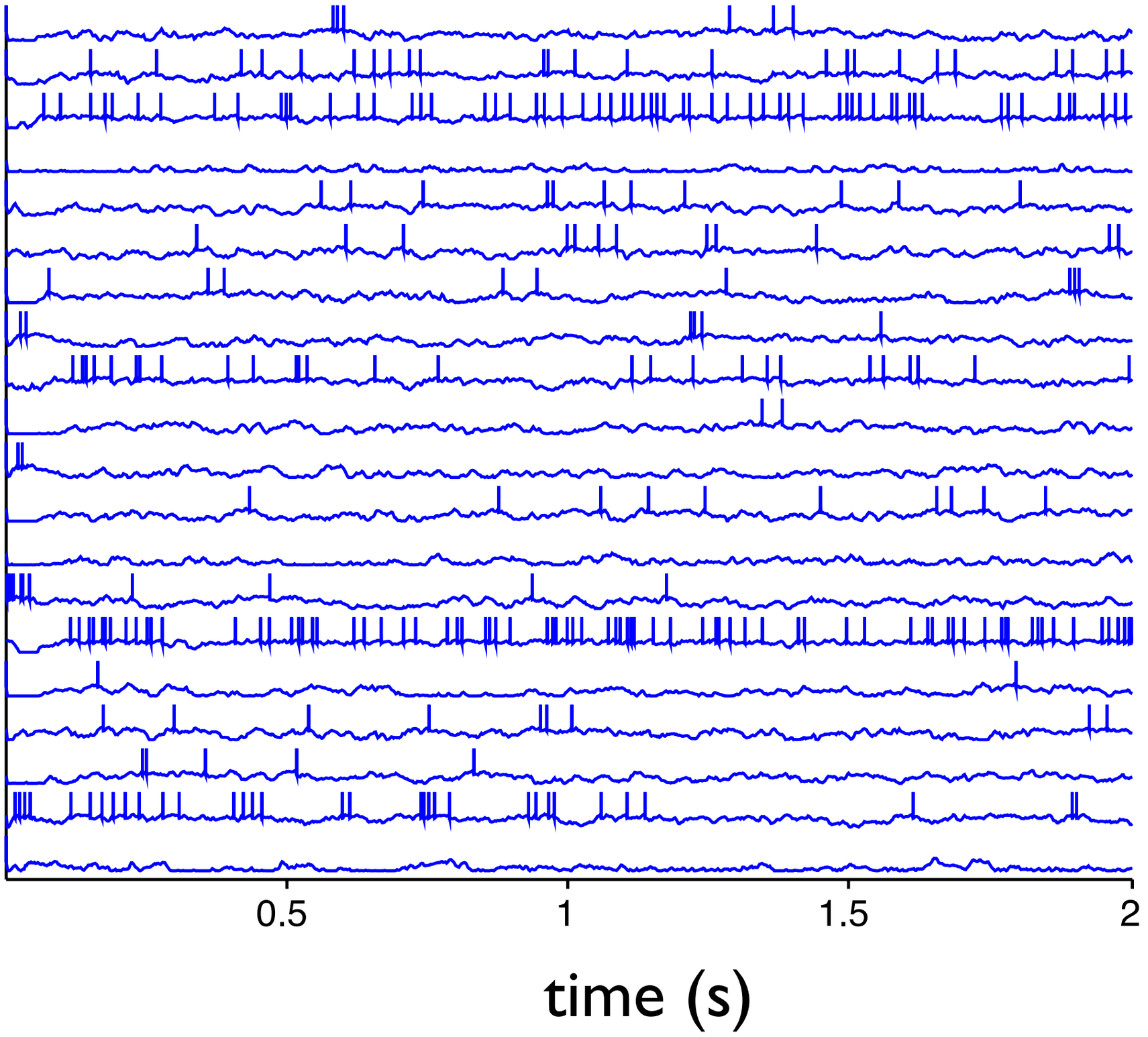
2

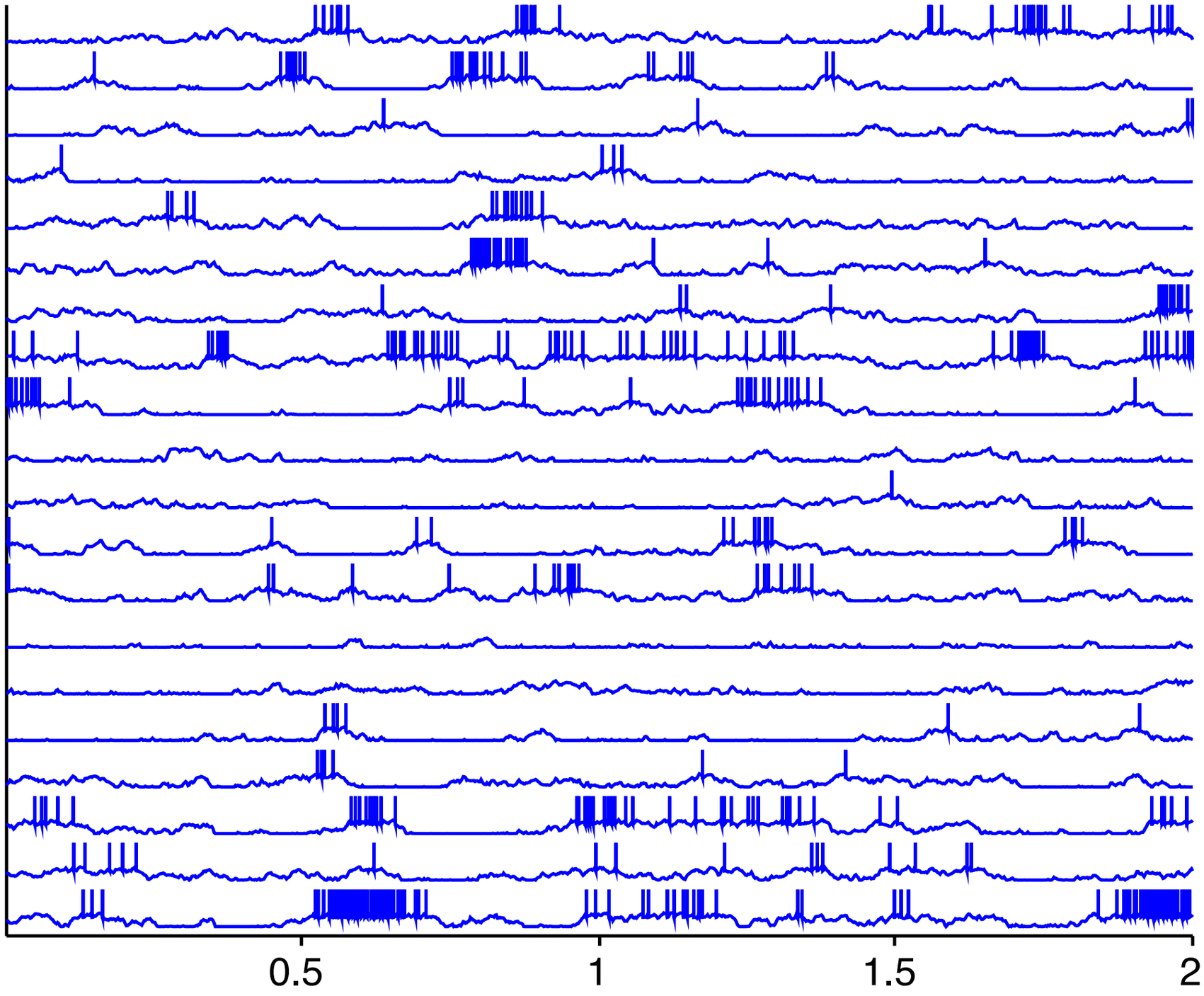
4

time (s)


1
2
3
4
5
V
1
2
3
4
5
time (s)


f^{in}
f
time (s)
1
2
3
4
5


0.5
1
1.5
2
time (s)


0.5
1
1.5
2
time (s)

input

output

1.0

neuron 1

neuron 2

neuron 3

Incorporating constraints on J^L

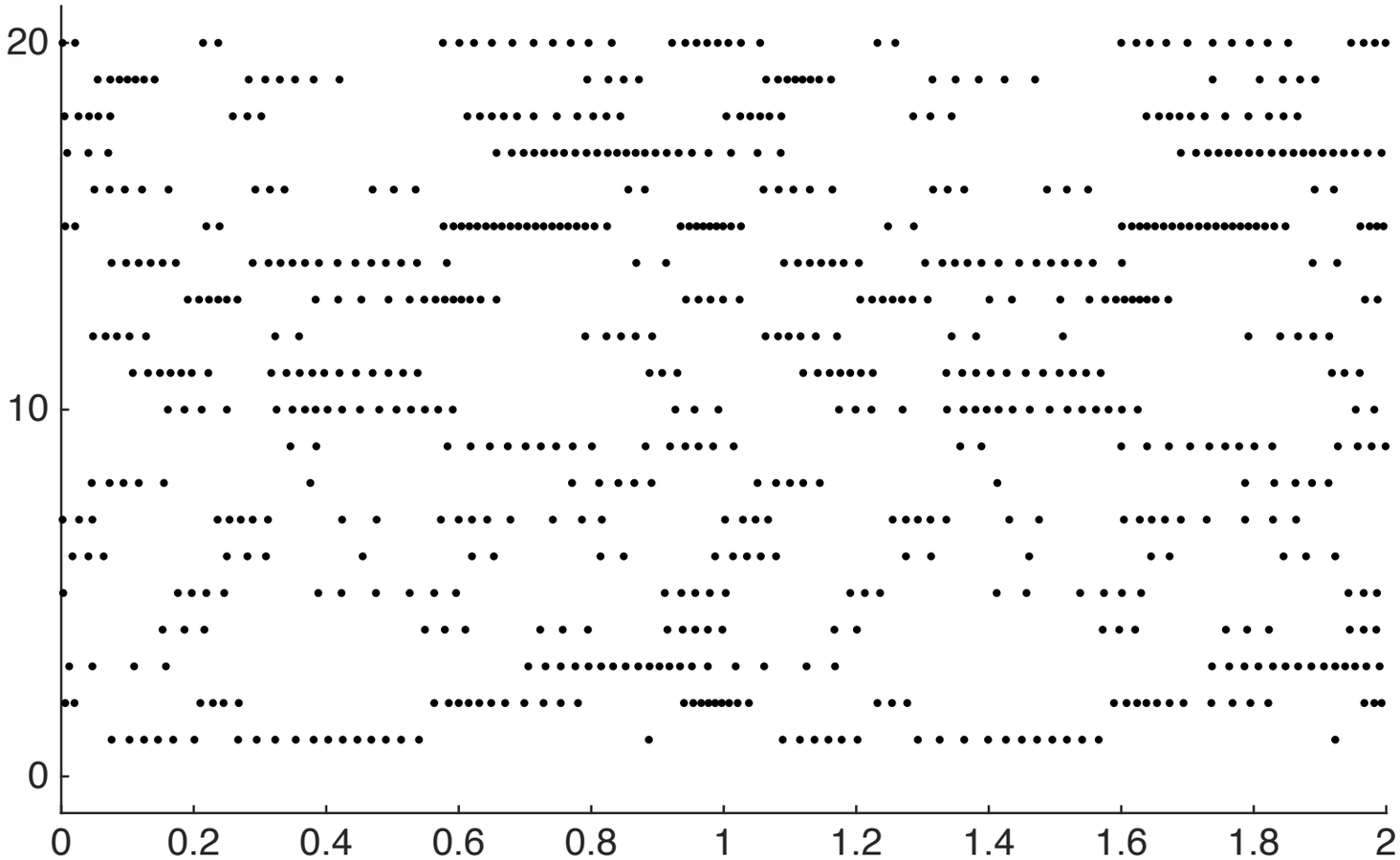
$$E = \int_0^T ||J^L s(t) - f(t)||^2$$

sparse connections

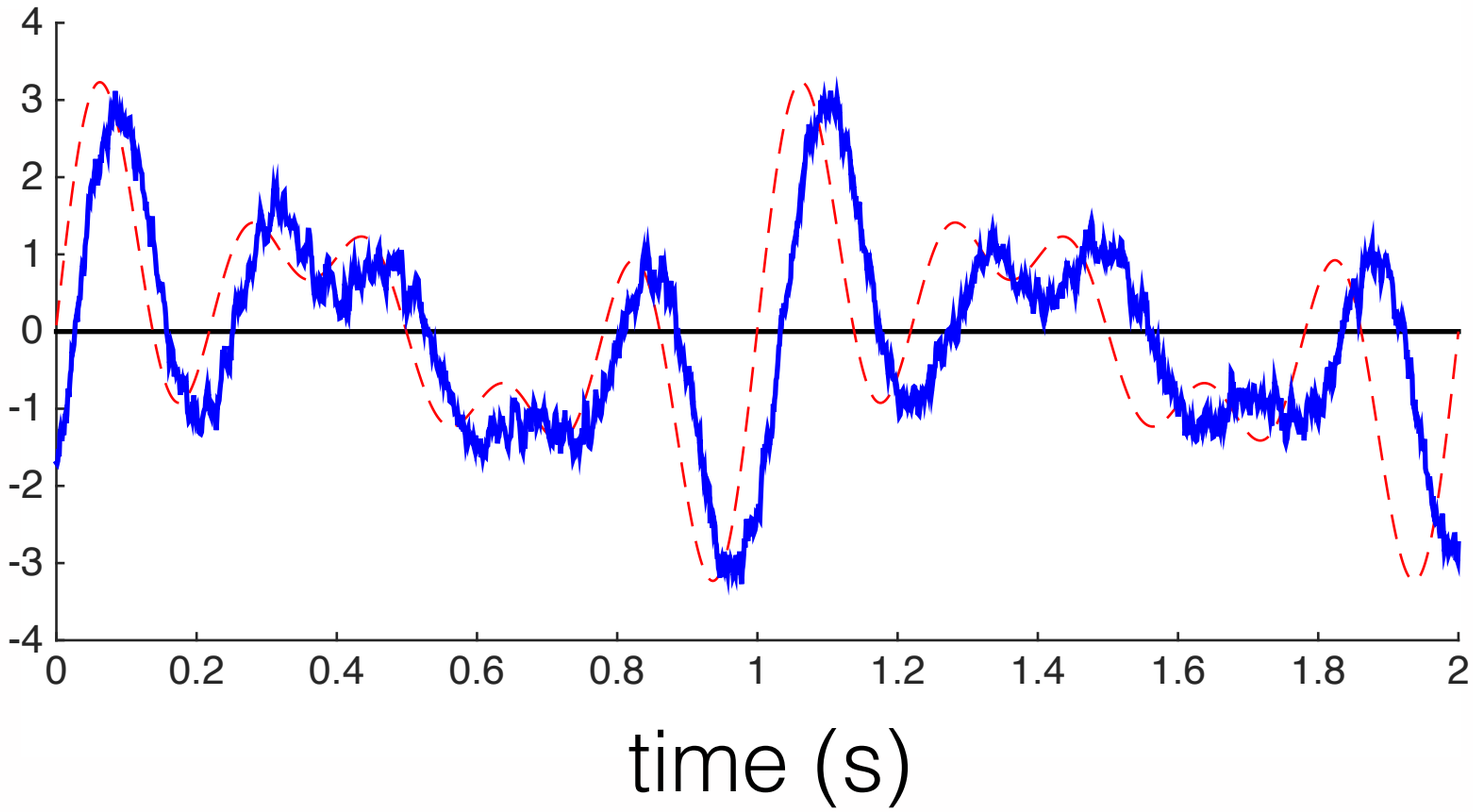
Dale's law

1000 N, 50% sparsity, Dale's Law

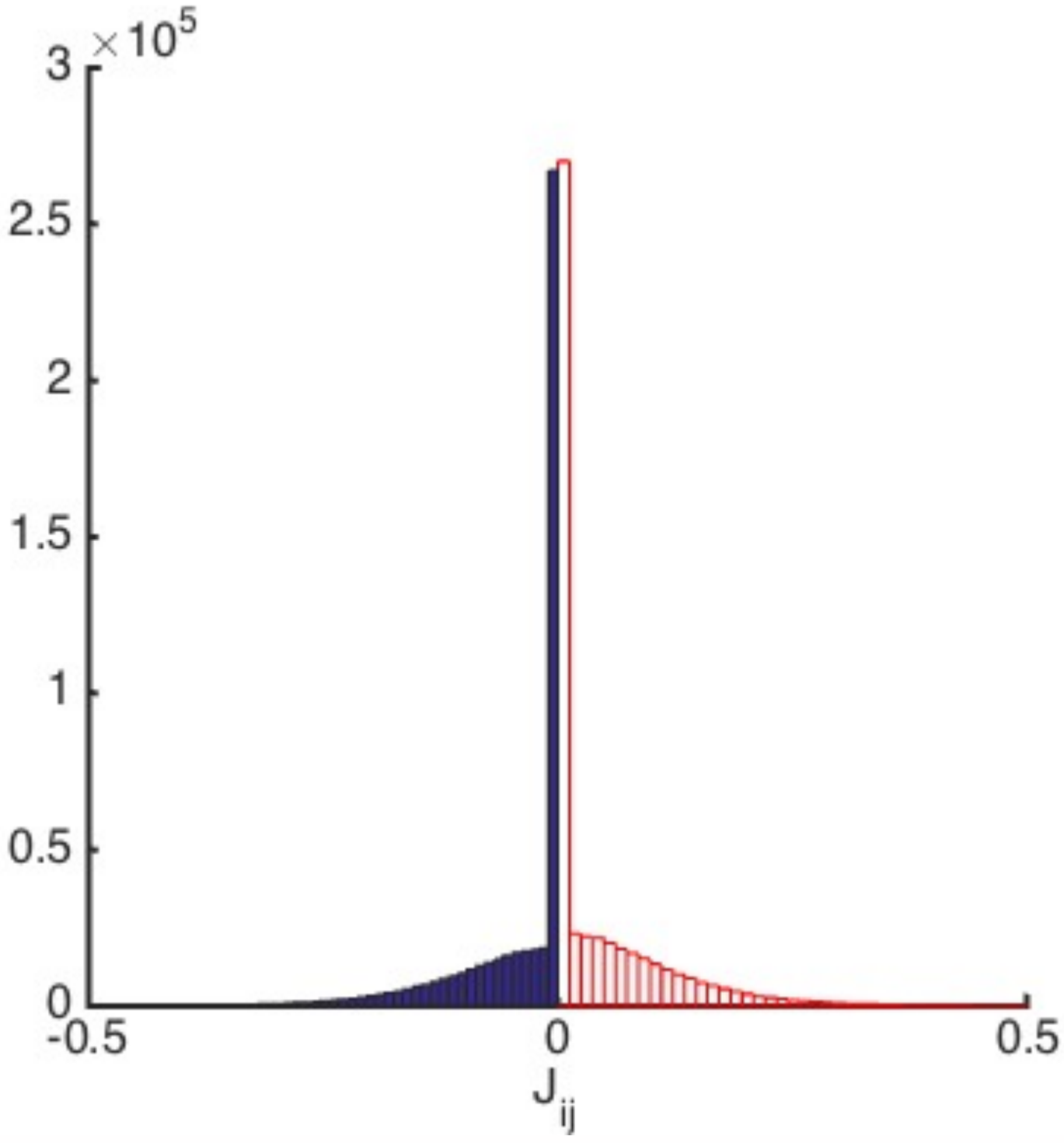
spikes

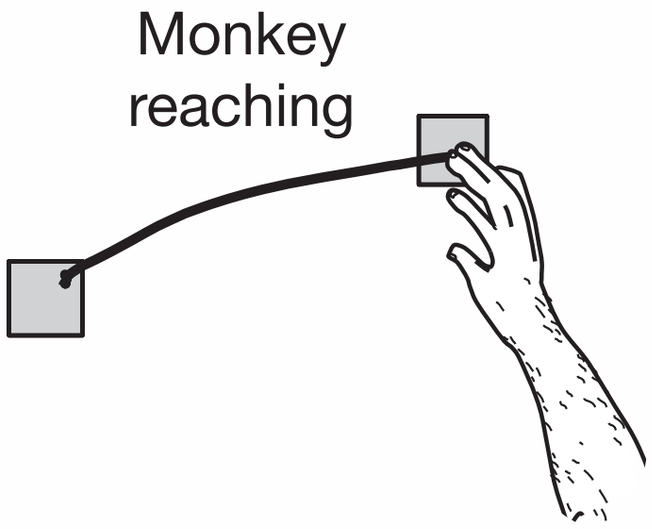


$Z(t)$



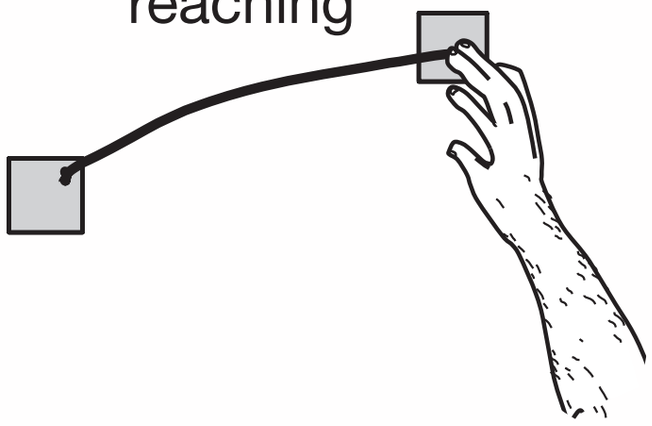
1000 N, 50% sparsity, Dale's Law



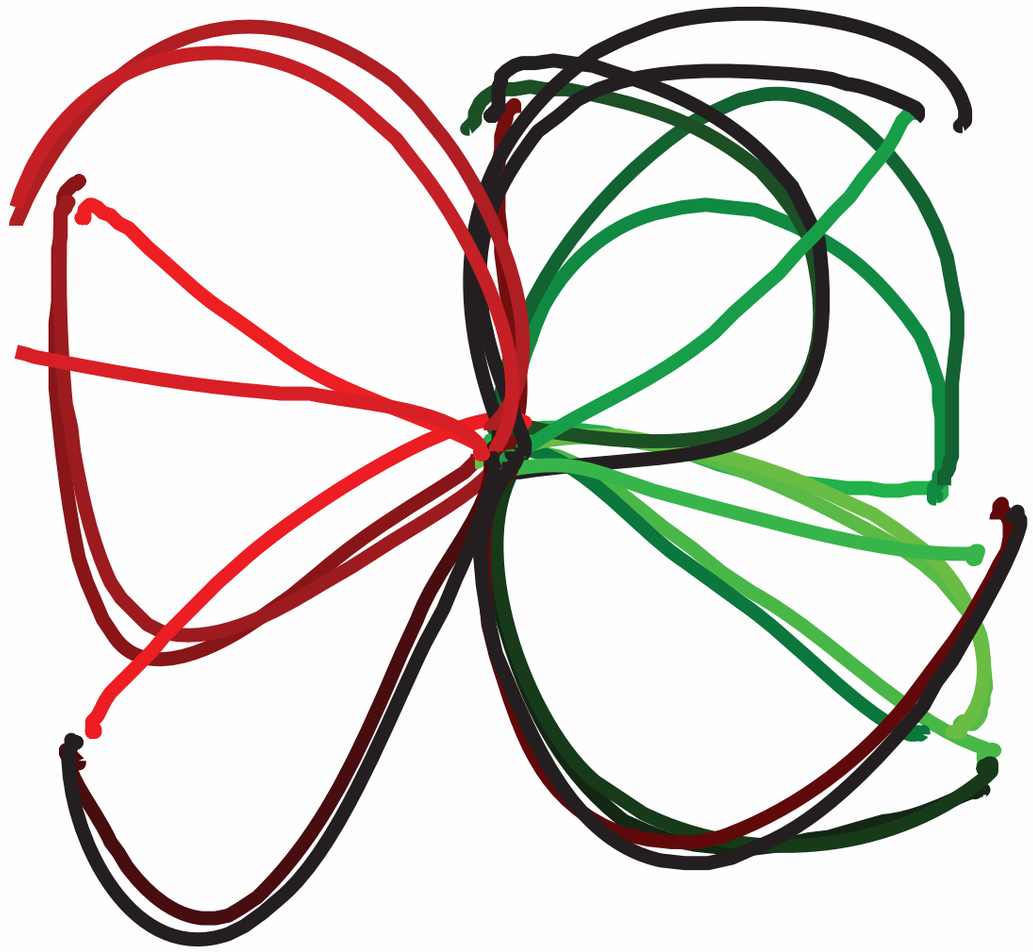


Churchland, Kaufman, Ryu, Shenoy

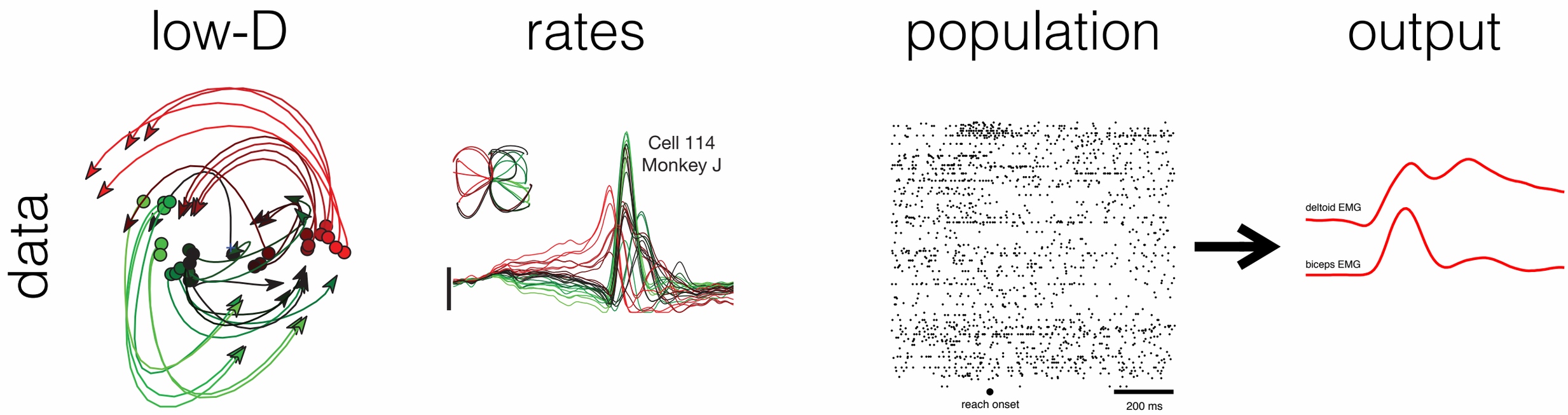
Monkey reaching



27 reach “conditions”



Churchland, Kaufman, Ryu, Shenoy



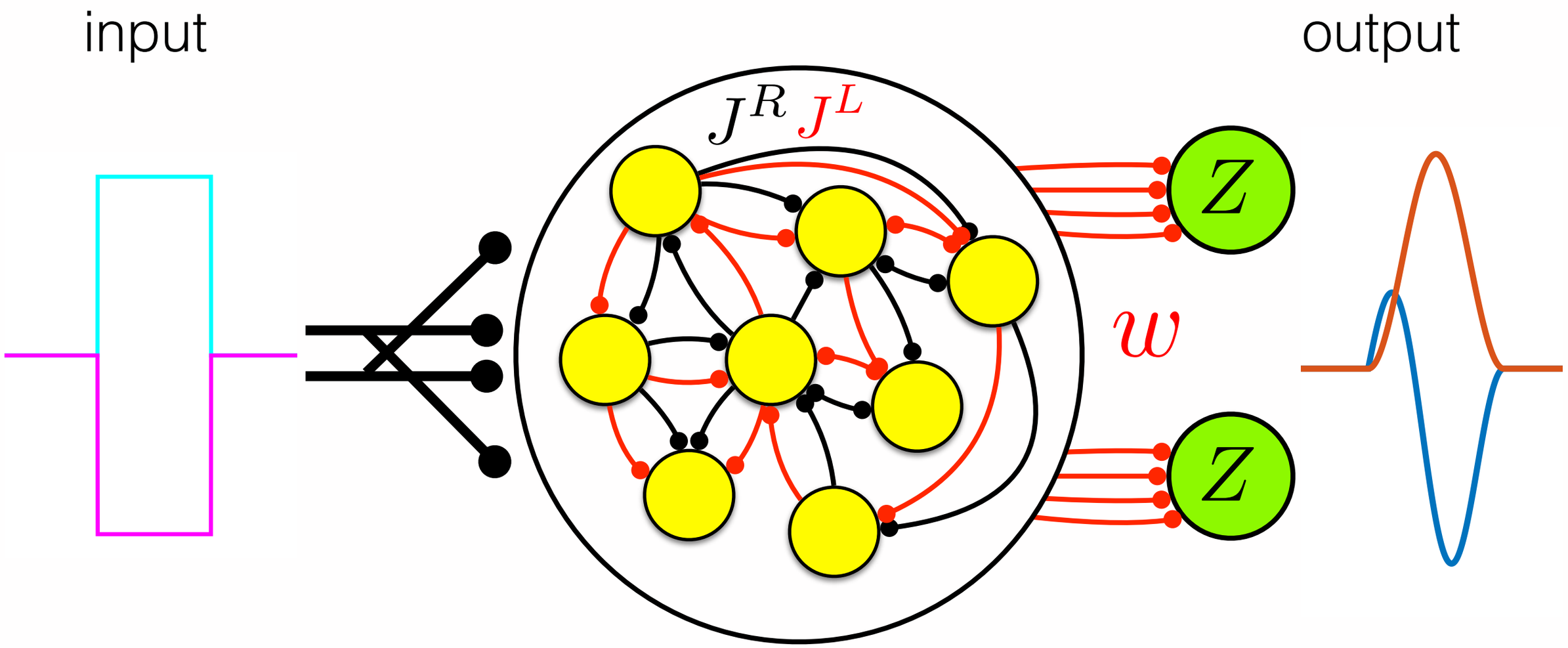
low-dimensional oscillatory projection across all conditions

heterogeneity across conditions for individual neurons

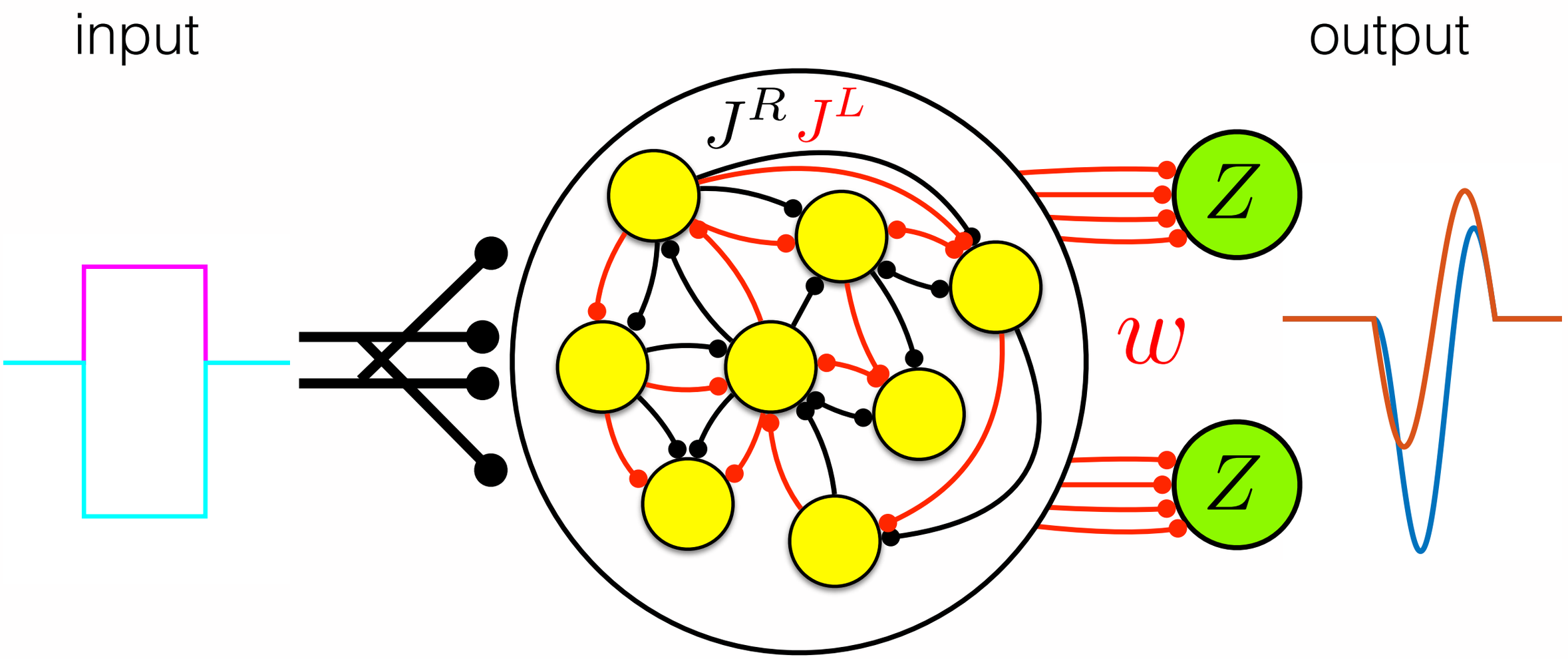
heterogeneous population response

smooth network output

Churchland, Cunningham et. al. 2012



condition 1

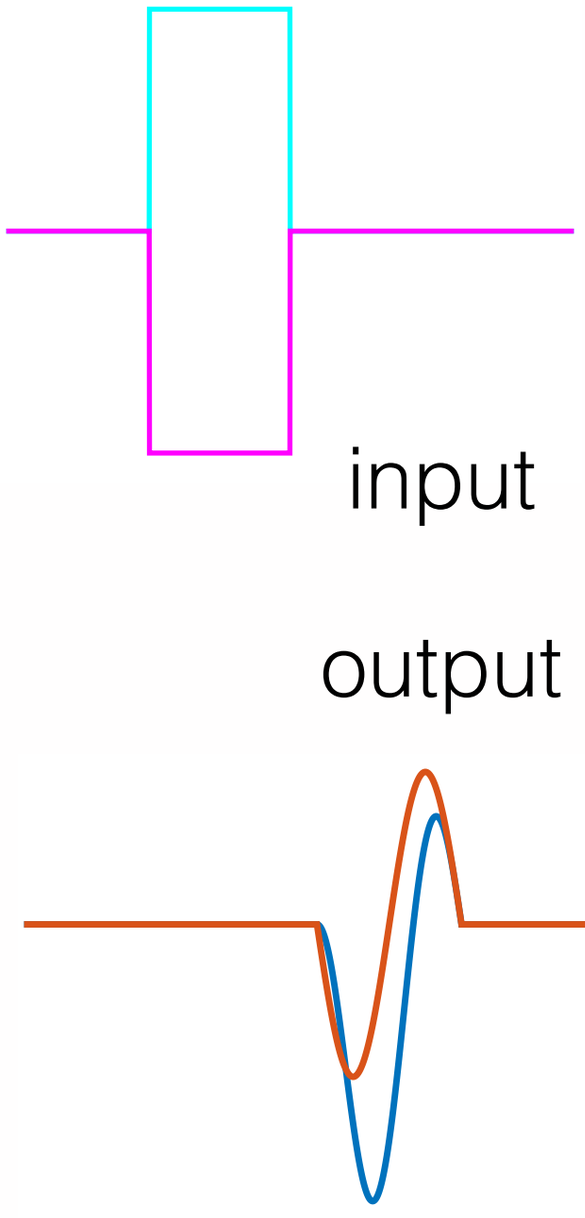


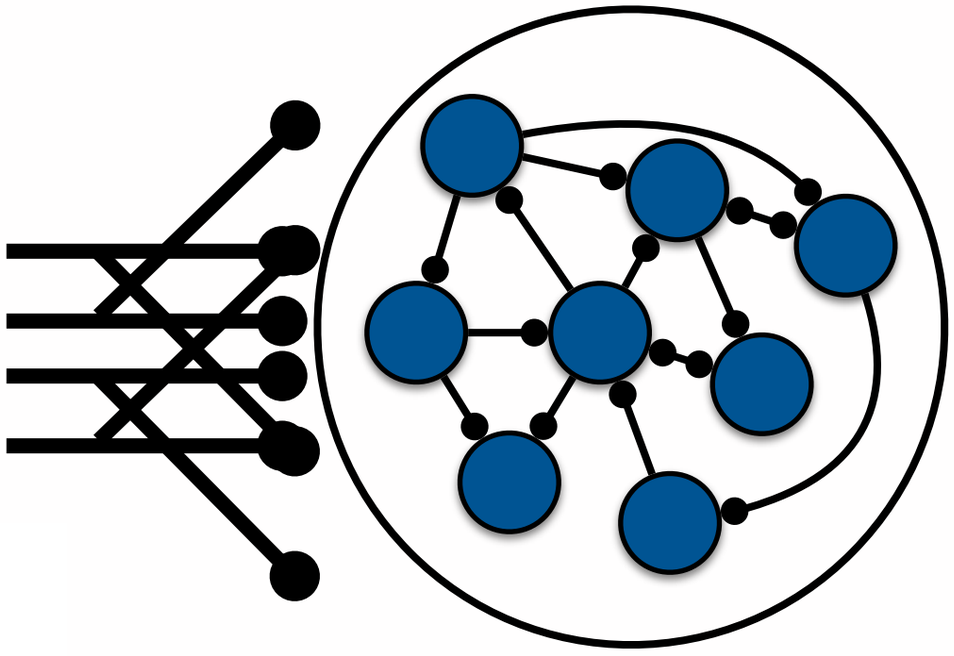
condition 2

⋮

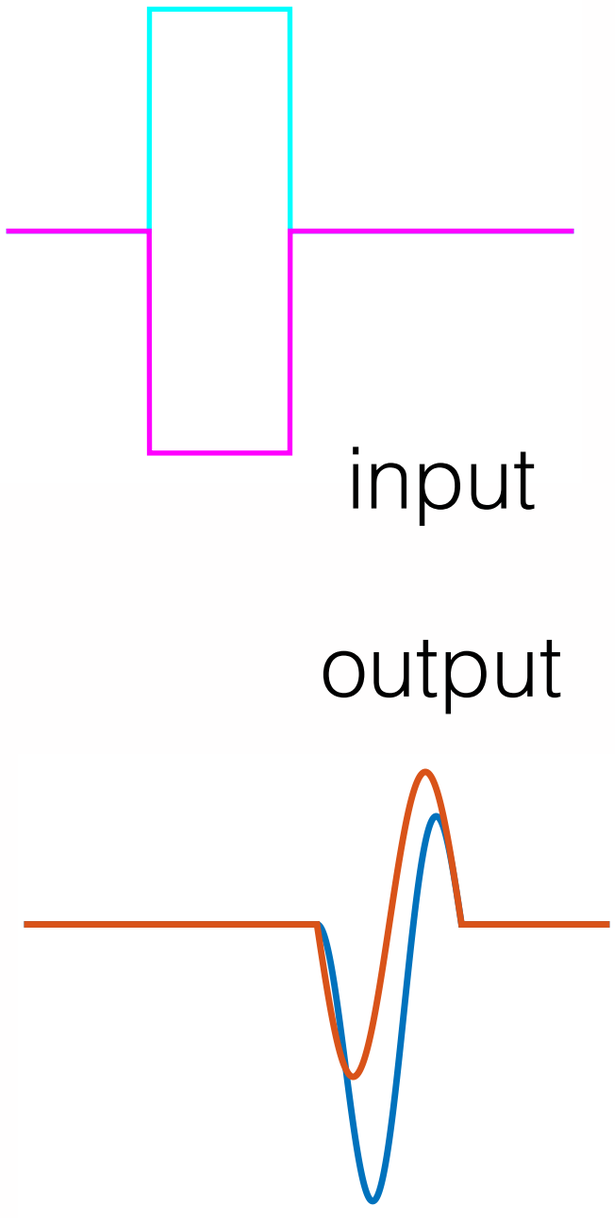
condition 8

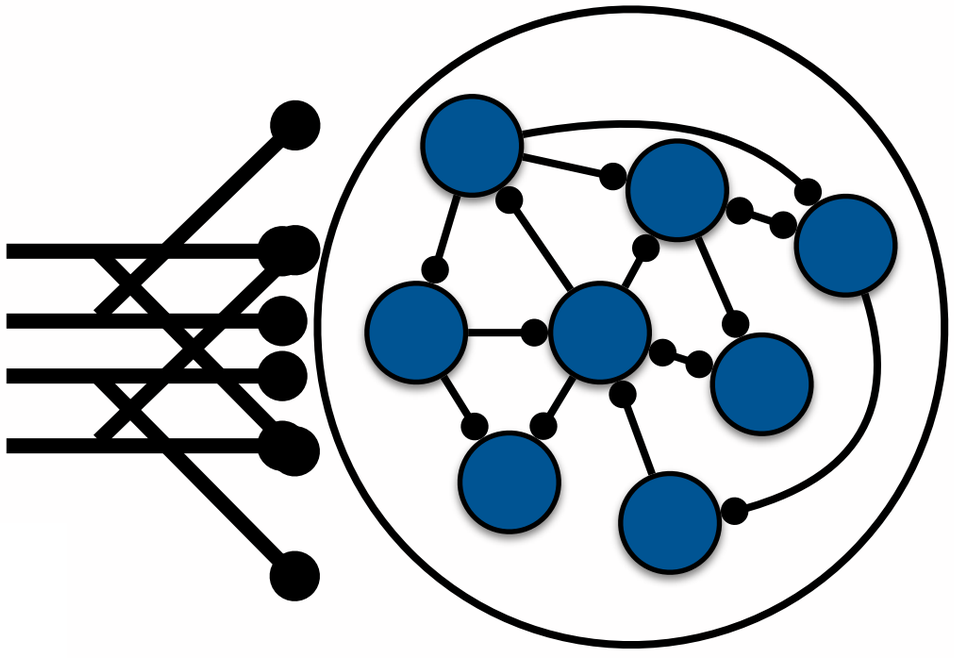
firing rate model

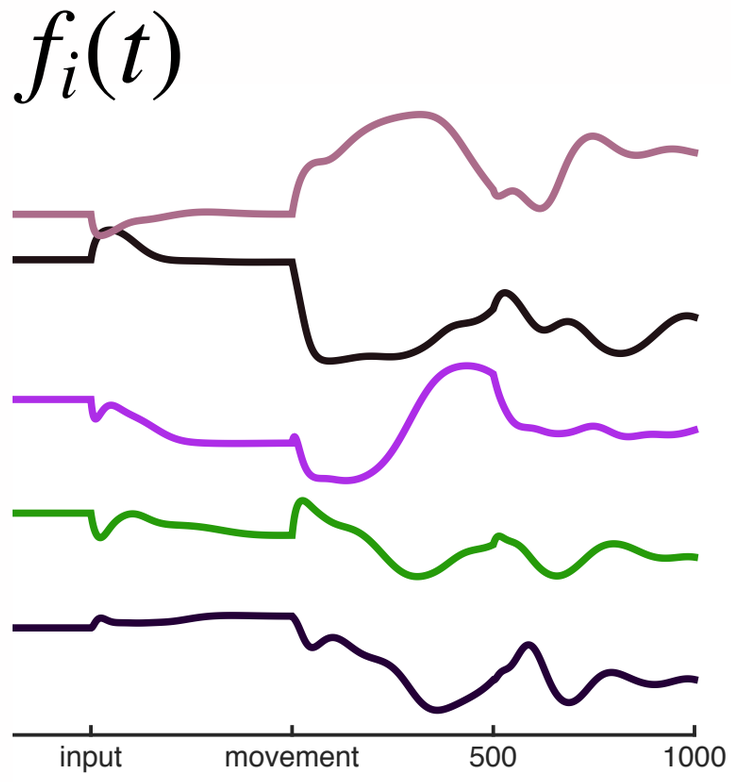


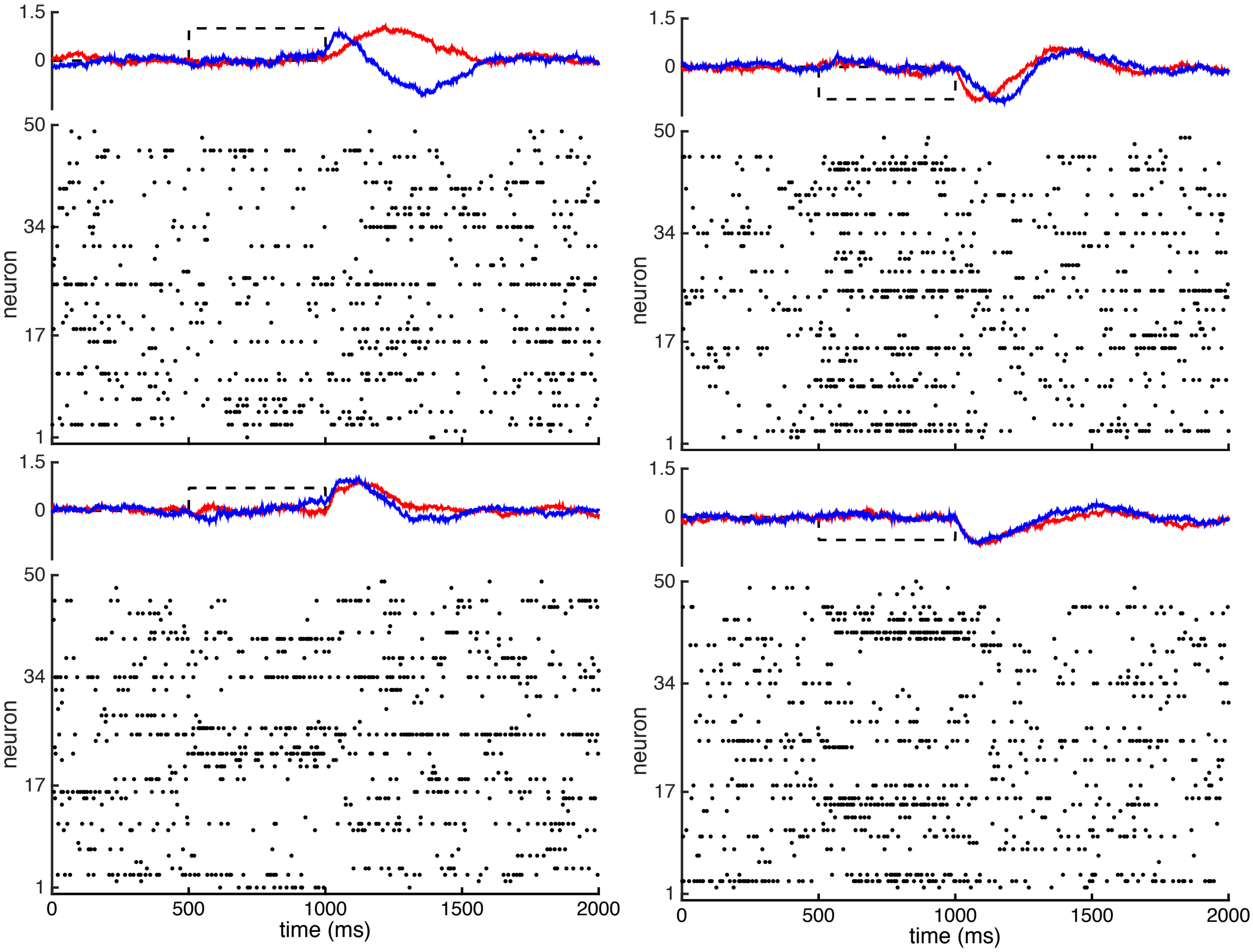


firing rate model








1.5
0
50
34
17
1
neuron
0
500
1000
1500
2000
time (ms)

“early movement”

“late movement”

“preparatory”

output

trials

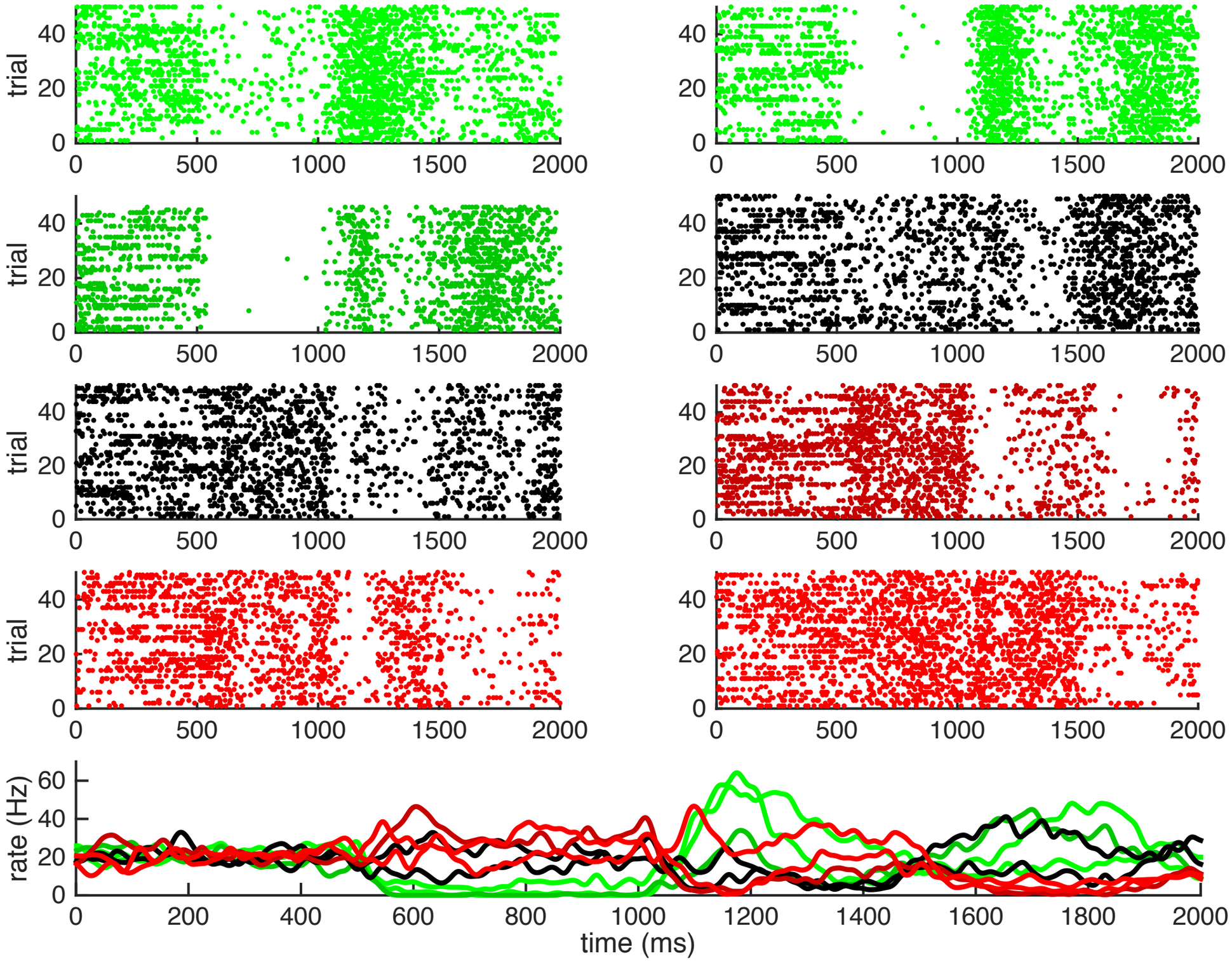
Firing rate (Hz)

time (ms)

count

fanofactor

1 neuron, 8 conditions



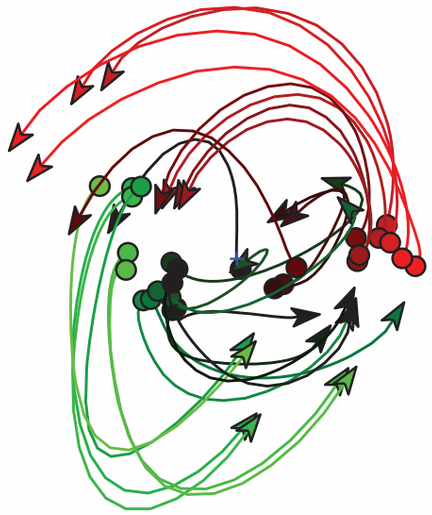
low-D

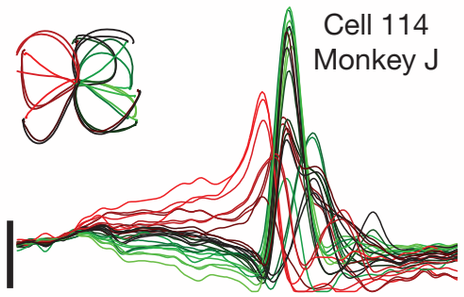
rates

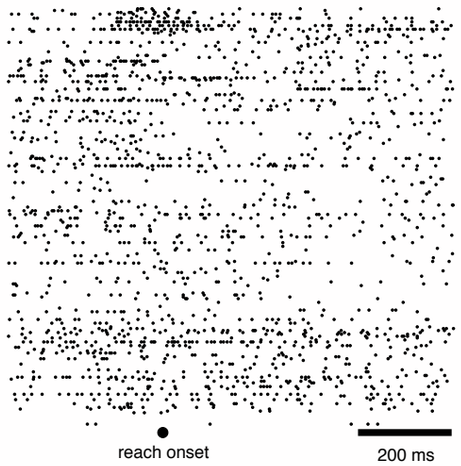
population

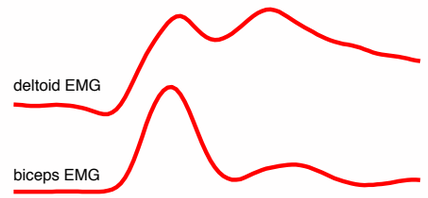
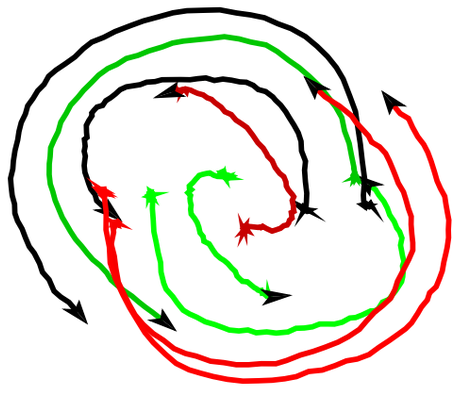
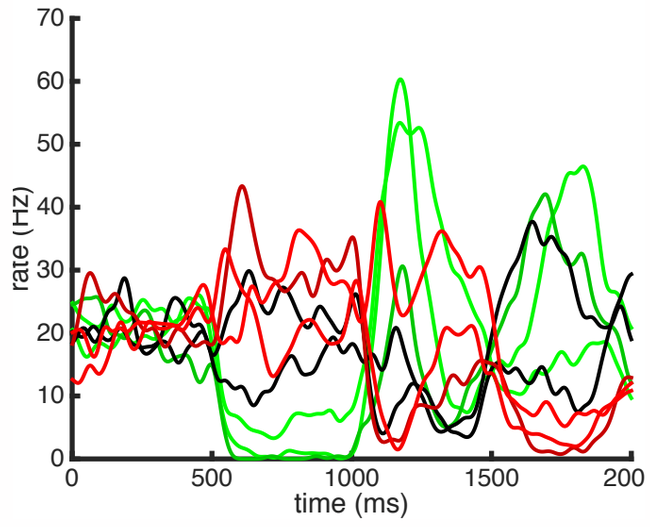
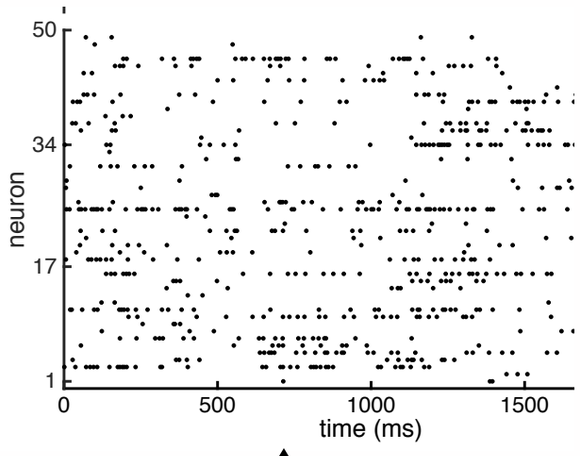
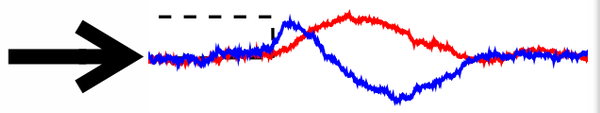
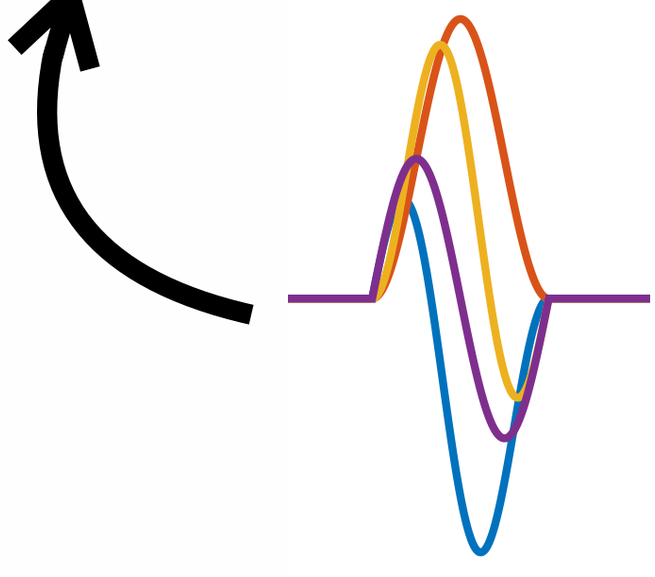
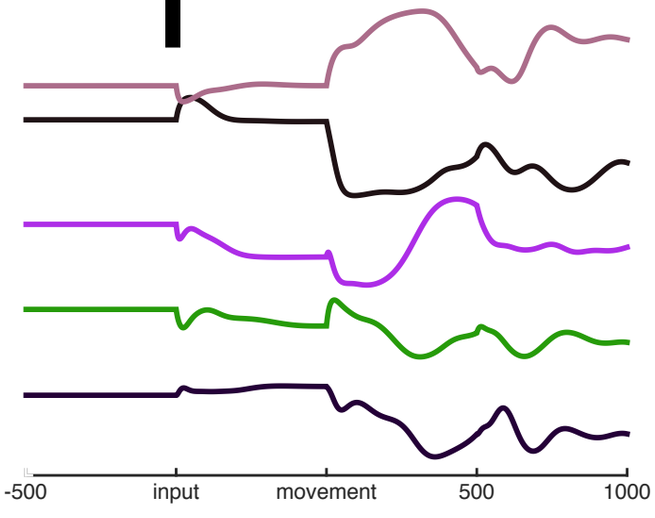
output

data









model